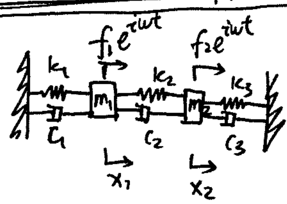
Ⅱ、2DOF

Complex number approach to 2DOF systems



$$\begin{pmatrix} m_1 & \\ & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2+c_3 \end{pmatrix}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} e^{iwt}$$

Note: Classical normal modes exist only when their is no damping.

Define: $[Z(s)] \equiv s^2[m] + s[c] + [k]$

Suppose $\underset{\sim}{x} = \underset{\sim}{a} e^{iwt}$ $(a \in \mathbb{C}^2)$,

$$\{[m](iw)^2 + [C](iw) + [k]\} \underset{\sim}{a} = \underset{\sim}{f}$$

$$\Rightarrow \underset{\sim}{a} = [Z(iw)]^{-1} \underset{\sim}{f} \qquad (1)$$

For homogeneous solutions, let $\underset{\sim}{x} = \underset{\sim}{\lambda} e^{st}$,

$$\{[m]s^2 + [c]s + [k]\} \underset{\sim}{\lambda} = 0$$

$$\Rightarrow [Z(s)] \underset{\sim}{\lambda} = 0$$

Let $\det[Z(s)] = 0$, we get $s = s_1, s_2, s_3$, or s_4, ~~with $\underset{\sim}{\lambda} = \underset{\sim}{\lambda_1}, \underset{\sim}{\lambda_2}, \underset{\sim}{\lambda_3}$, and $\underset{\sim}{\lambda_4}$ resp.~~ (2)

~~The complete solution is:~~

$$\cancel{\underset{\sim}{x}(t) = \sum_{j=1}^{4} \underset{\sim}{\lambda_j} e^{s_j t} + [Z(iw)]^{-1} \underset{\sim}{f} e^{iwt}}$$

Define $r_j \equiv \dfrac{\lambda_{2j}}{\lambda_{1j}}$,

From $[Z(s_j)] \underset{\sim}{\lambda_j} = 0$, we get $r_j = -\dfrac{Z_{11}(s_j)}{Z_{12}(s_j)}$ (3)

$\underset{\sim}{x}(t)$ has the form :

$$\underset{\sim}{x}(t) = \sum_{j=1}^{4} \lambda_{1j} \begin{pmatrix} 1 \\ r_j \end{pmatrix} e^{s_j t} + \underset{\sim}{a}\, e^{i\omega t} \qquad (*)$$

With I.C.'s $\quad \underset{\sim}{x}(0) = \underset{\sim}{x}_0 \,, \quad \dot{\underset{\sim}{x}}(0) = \dot{\underset{\sim}{x}}_0$, we have

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ r_1 & r_2 & r_3 & r_4 \\ s_1 & s_2 & s_3 & s_4 \\ r_1 s_1 & r_2 s_2 & r_3 s_3 & r_4 s_4 \end{pmatrix} \begin{pmatrix} \lambda_{11} \\ \lambda_{12} \\ \lambda_{13} \\ \lambda_{14} \end{pmatrix} = \begin{pmatrix} x_{01} - a_1 \\ x_{02} - a_2 \\ \dot{x}_{01} - i\omega a_1 \\ \dot{x}_{02} - i\omega a_2 \end{pmatrix} \qquad (4)$$

Solving the equation gives λ_{1j}.

Hence, the complete solution is $(*)$, with parameters determined by (1) ~ (4).