

<<多孔介质中的渗流物理>>

导论:

第一章第二章: 多孔介质和流体的性质; 第三章, 多孔介质与流体的相互作用 (静力学);
第四~九章: 多孔介质与流体相互作用 (单相流体的水动力学); 第十~十一章,
多孔介质与流体相互作用 (若干流体的水动力学)。

第一章 多孔介质

1.1 多孔介质的种类和几何特性

多孔介质: 1° (描述性定义) 内部含有孔隙的固体。

2° (举例) 砂砾堆、填料, 由颗粒铺成的层状物, 石灰岩、白云岩、
泡沫岩等多孔岩石, 棉布、毛毡、滤纸等纤维性聚合物,
含微孔隙的催化剂

孔隙: 1° (直观定义) 物料中空而无物的间隙

2° (限制) 介于洞穴和分子间隙的大小之间的空隙。

多孔的: 有很多孔隙的物料被称为多孔的。

多孔介质的有效孔隙度: $\frac{\text{多孔体系中彼此连通的孔隙}}{\text{总体积}}$

Manegold 的孔隙空间分级法:

① 空隙: 内壁对水动力学动向影响不大的孔隙。

② 毛管隙: 内壁对水动力学动向影响很大的孔隙。

③ 迫开隙: 内壁可以影响流体分子结构的孔隙。

$$\text{孔隙度 } P = \frac{\text{孔隙}}{\text{总体积}}$$

$$\text{空隙比 } e = \frac{P}{1-P}$$

$$\text{比内表面积 } S = \frac{\text{内表面积}}{\text{外表体积}}$$

孔隙直径 ^{δ} : 孔隙空间中任何一点, 称在该点装得进去且整个都在孔隙空间中的最大球形直径。

孔隙大小分布 $\alpha(\delta)$: 孔隙直径在 δ 与 $\delta+d\delta$ 之间的孔隙占总孔隙空间的比率。

注: 孔隙大小分布有性质 $\int_0^{\infty} \alpha(\delta) d\delta = 1$

累积的孔隙大小分布 $f(\delta)$: 孔隙直径大于 δ 的孔隙占总孔隙空间的比率。

$$f(\delta) = \int_{\delta}^{\infty} \alpha(\delta) d\delta$$

$$\text{注: } \frac{df}{d\delta} = -\alpha$$

迂曲度 T : $\frac{\text{一个流体质点流程的平均长度}}{\text{多孔介质两截面间长度}}$

在多孔介质的某一截面上, 随意勾划出一条可以求长的线段。线段上的点可以用弧长 S 标记。定义 $f(S) = \begin{cases} 1, & \text{点 } S \text{ 在空隙空间} \\ -1, & \text{点 } S \text{ 在固相空间} \end{cases}$

其平均值 $\hat{f} = \lim_{S \rightarrow \infty} \frac{\int_{-S}^S f(s) ds}{\int_{-S}^S ds}$ 。 \hat{f} 与孔隙度 P 有线性关系: $\hat{f} = 2P - 1$ 。

定义 $f^*(S) = f(S) - (2P - 1)$ 。

自相关函数 $R^*(\Delta) = \lim_{S \rightarrow \infty} \frac{\int_{-S}^S f^*(s) f^*(s+\Delta) ds}{\int_{-S}^S ds}$

自相关函数 $R^*(\Delta)$ 可以用于表征多孔介质的特征。

1.2 孔隙度的测量

1° 直接法: 压实, 适于十分松软的多孔材料。

2° 光学法: 认为多孔介质某一随机切片的平面孔隙度与多孔介质的孔隙度相同。前者可以通过显微照相测得。

注: 1° 如果多孔介质是松散的, 可用石蜡、塑料等浸渍, 再切片。可以加入染料增强对比度。

3° 密度法: 已知构成多孔介质的材料的密度为 ρ_G , 测得多孔介质的表观密度为 ρ_B . 则 $P = 1 - \frac{\rho_B}{\rho_G}$.

表观密度的测量法: 体积排替法 ~~的~~ —— 假设不润湿流体, 如水银, 不会进入多孔介质. 则可以测得多孔介质的外表体积.

构成多孔介质的材料的密度的测量法: 1° 捣碎, 排替法
2° 浸渍法 (测量多孔介质在浸透液体前后的重量变化)

4° 气体膨胀法 直接测量孔隙空间中所含气体的体积.

测量气体、多孔介质混合系统的体积与压力, 测量多孔介质的外表体积, 依据 $P_1 V_1 = P_2 V_2$ 可求得孔隙体积.

优点: 快速、准确、试样不受损.

1.3 比面 (比内表面积) 的测量

1° 光学法: 测量切片的显微照片上孔隙周长与切片总面积.

假设与 1.2 的光学法相似.

2° 以吸附作用为依据的方法:

3° 以渗流为依据的方法

4° 其它

1.4 确定孔隙大小分布情况

注水银法;

表面吸附作用;

X-射线散射法;

光学法;

孔隙大小分布的一般表达式:

$$\frac{df}{d\delta} = \frac{k}{4} \operatorname{sech}^2 \left[\frac{k}{2} (\delta - \delta') \right]$$

其中, δ' 是最可几孔隙直径, k 是个参数.

1.5 颗粒大小与孔隙结构间的对比关系。

非胶结多孔介质的粒度大小分布也可用于研究孔隙特性。

1.6 迂曲度的测量

$$\text{水流的迂曲度 } T_{hy} = \frac{P}{k} \frac{A_{eff}}{8\pi}$$

其中, k 为渗透率, A_{eff} 为有效截面积。

$$\text{电流的迂曲度 } T_{el} = PF$$

其中, 地层因素 $F = \frac{\rho_0}{\rho_w}$, ρ_0 为多孔介质 100% 饱和着流体时的电阻率, ρ_w 为流体电阻率。

假设水流通道与电流通一致, 或者说 T_{hy} , T_{el} 一致, 则可用上述电阻测量法测量迂曲度。

1.7 多孔介质的流变学特性

多孔介质中的应力状况:

Terzaghi 的理论:

多孔介质受到的应力分为中性平衡应力和有效应力, 有效应力使多孔介质发生形变。

有效应力的定义:

$$\textcircled{1} \text{ 微观 } \tau_{ik} = \tau_{ik}^m (1-P) + p \delta_{ik} P$$

其中, τ_{ik} 为应力张量, τ_{ik}^m 为有效应力张量

$$\textcircled{2} \text{ 宏观 } \tau_{ik} = \tau_{ik}^m + p \delta_{ik} P$$

$$\textcircled{3} \text{ 虚拟 (Gershanov) } \tau_{ik}^F = \tau_{ik}^m - p \delta_{ik} (1-P)$$

$$\text{或 } \tau_{ik} = \tau_{ik}^F + p \delta_{ik}$$

多孔介质的形变:

总可以用应力张量 τ 的某些分量作为孔隙度的参变量:

$$P = P_0 \tau(p)$$

常用形式为 $P = P_0 (\gamma - p)^{-c}$

其中, γ 为多孔介质的总压力标量, p 为流体中的压力。

对于各向异性的应力, 可以引入参量 ξ_x , 它是多孔介质在任意状态下的笛卡尔坐标。记点 ξ_a 在受力状态下的座标为 $X_i(\xi_a)$, 则

$$X_i = X_i(\xi_a, \tau_{ik}, p, P)$$

$$\xi_a = \xi_a(X_i, \tau_{ik}, p, P)$$

这表征着多孔介质的形变。

压实作用:

假设流体压力恒为 p , 则应力张量与孔隙度间的微分关系为:

$$\frac{dP}{d\gamma} = -\mu P$$

其中, μ 是一个压实作用系数, 它现在仅取决于孔隙度和状态压力 γ 。

对于粘土, μ 、 γ 、 P 间关系为

$$\mu = \frac{\ln \frac{P_0}{P}}{\gamma - 1}$$

多孔介质的破裂情况:

除了一般固体所见到的破裂模式外, 孔隙流体会导致一种特殊形式的破裂, 称为劈裂性破裂。它是由流体压力引起的多孔结构的拉伸破坏。

第二章 流体

2-1 概述

2-2 连续介质理论

① 流变条件 (本构关系): 应力、应变, 或应力率、应变率间关系。

一般方程为
$$\rho = \rho_0 \left(\frac{p}{p_0}\right)^m e^{\beta_f (p - p_0)}$$

其中, 常数 m 、 β_f 的取值如下:

		m		β_f	
液体		0	不可压	0	
			可压缩	$\neq 0$	
气体	等温	1	0		
	绝热	$\frac{C_v}{C_p}$			

② 对非牛顿流体, 在流体平行于管壁流动时, Bingham 条件为:
(一种本构关系)

$$\tau - \tau_y = \mu \frac{dv}{dr}$$

其中 τ_y 是屈服强度。

对于管子, 截面平均流速 \bar{v} 与管壁处剪力满足关系:

$$\frac{8\bar{v}}{D} = \frac{\tau_w}{\mu} \left[1 - \frac{4}{3} \frac{\tau_y}{\tau_w} + \frac{1}{3} \left(\frac{\tau_y}{\tau_w}\right)^4 \right]$$

其中, D 为管子直径。

介于牛顿流体与宾哈姆关系间的拟塑性流^{一般}动的经验公式为:
(另一种本构关系)

$$\tau = \mu \frac{dv}{dr} + \frac{1}{B} \sinh^{-1} \left(\frac{1}{A} \frac{dv}{dr} \right)$$

2-3 分子渗流

毛管流量公式 $Q = \frac{4}{3} \sqrt{\frac{2\pi RT}{M}} \frac{a^3}{h} \frac{\Delta p}{P}$, a 为半径, h 为长度, M 是分子量。
(Knudsen 方程)

由气体分子平均自由程的公式 $\lambda = c \frac{\mu}{P} \sqrt{\frac{RT}{M}}$, \bar{p} 为平均压力。

得 Knudsen 方程的另一形式

$$Q = \frac{4}{3} \sqrt{2\pi} \frac{\lambda}{c\mu} \frac{a^3}{h} \bar{p} \frac{\Delta p}{P}$$

(气体) 分子平均自由程 λ 与管半径 a 在同一量级时, 泊肃叶公式成立;

λ 远大于 a 时, Knudsen 方程成立;

λ 介于上述两种情况之间时, 流量公式可写为

$$Q = \frac{\pi a^4 \Delta P}{8\mu h} \frac{\bar{P}}{P} + \varepsilon \cdot \frac{4}{3} \sqrt{\frac{2\pi RT}{M}} \cdot \frac{a^3}{h} \frac{\Delta P}{P}$$

对于任何有意义的气流状况, ε 是常数, 称为 Adzumi 常数。

2.4 流体与表面的相互作用。

- ① 在离管壁约若干个分子的距离内, 管壁和流体分子之间有一个强引力位势。从而, 器壁的流体分子浓度比容器内的浓度大, 这种现象叫吸附作用。吸附作用中, 器壁会释放能量, 称为吸附热。

吸附作用等温线: 容器内的分子压力与容器壁上的分子数的关系曲线, 吸附过程保持等温。

② 界面张力

界面张力: 液固或液气之间, 由于物质内部分子对其表面上分子的固有引力产生的自由表面能。

表面张力: 物质与真空之间的界面能。

界面张力与表面张力均以 γ 表示, 用下标 S、L、A 标记固体、液体、气体。

界面张力用双下标, 表明物质与其参考系; 表面张力用单下标。

把固体上单位面积的液体揭开需要的功 W_{SL} 满足:

$$W_{SL} = \gamma_S - \gamma_L - \gamma_{SL}$$

接触角 θ : 两种流体相在一点与固体接触, 平衡时, 流体相间界面与固体形成的角。

~~对于液气界面~~

对于液气界面, θ 满足

$$\cos \theta = \frac{\gamma_{SA} - \gamma_{SL}}{\gamma_{LA}}$$

液气界面面积可认为是视表面积，固液界面面积则比视表面积大得多，为视表面积的 σ 倍。其中， σ 为固体的“表面粗糙度”。

视接触角 θ' 满足 $\cos \theta' = \sigma \cos \theta$ 。

2.5 毛管系中的多相流动

① 不混溶流体

② 混溶流体

两种流体混合的速度取决于分子扩散作用的速度与对流速度。

对流作用：对于圆管，管内流速分布为 $V = V_0(1 - \frac{r^2}{a^2})$

截面上平均浓度 $C_m = \frac{2}{a^2} \int_0^a C r dr$

1° 若在 $x=0$ 与 $x=X$ 间充满着浓度为 C_0 的可溶物，当流动沿轴向，所以在 r 到 $r+\delta r$ 间的可溶物量不变。（ $X \ll a$ ）



可溶物分布为 - 抛物面 $X = V_0 t (1 - \frac{r^2}{a^2})$ ，其径向厚度不变，为 X 。

在筒面 r 到 $r+\delta r$ 间的可溶物的量为 $2\pi r \delta r X C_0$ ，

（在 $0 \leq X \leq V_0 t$ 时）截面上可溶物的厚度 δr 与 δX ，即 X 之间的关系为 可溶物径向分布厚度

$$\frac{\delta r}{\delta X} = \left| \frac{dr}{dX} \right| = \frac{a^2}{V_0 t - 2r}$$

$$\therefore \text{截面平均浓度 } C_m = \frac{2\pi r \delta r X C_0}{\pi a^2 \delta X}$$

$$= \frac{C_0 X}{V_0 t}, \text{ 是个常数.}$$

$$\therefore C_m = \begin{cases} \frac{C_0 X}{V_0 t} & (0 \leq X \leq V_0 t) \\ 0 & (X > V_0 t \text{ 或 } X < 0) \end{cases}$$

2° 若在初始时， $x < 0$ 内全是可溶物， $x > 0$ 内全是溶剂。这与无穷多情形 1° 累积起来等价，从而截面平均浓度在 $[0, V_0 t]$ 内应线性分布。

$$C_m = \begin{cases} C_0 & (x < 0) \\ C_0(1 - \frac{x}{V_0 t}) & (0 \leq x \leq V_0 t) \\ 0 & (x > V_0 t) \end{cases}$$

扩散作用：假设浓度分布轴对称，则扩散方程化为

$$D \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial x^2} \right) = \frac{\partial C}{\partial t} + v_0 \left(1 - \frac{r^2}{a^2} \right) \frac{\partial C}{\partial x}$$

假定扩散系数 D 与 C 无关。

假设 $\frac{\partial^2 C}{\partial x^2} \ll \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r}$ ，记 $z = \frac{r}{a}$ ，则扩散方程化为

$$\frac{\partial^2 C}{\partial z^2} + \frac{1}{z} \frac{\partial C}{\partial z} = \frac{a^2}{D} \frac{\partial C}{\partial t} + \frac{a^2 v_0}{D} (1 - z^2) \frac{\partial C}{\partial x}$$

管壁不可透的边界条件为

$$\left. \frac{\partial C}{\partial z} \right|_{z=1} = 0$$

假设对流速度明显地慢于浓度径向扩散的速度，则上述方程可以得出一个近似解。

考虑一个以常速度 $\frac{v_0}{2}$ 运动的平面，记 $x_1 = x - \frac{1}{2} v_0 t$ ，则方程改写为

$$\frac{\partial^2 C}{\partial z^2} + \frac{1}{z} \frac{\partial C}{\partial z} = \frac{a^2}{D} \frac{\partial C}{\partial t} + \frac{a^2 v_0}{D} \left(\frac{1}{2} - z^2 \right) \frac{\partial C}{\partial x_1}$$

由于穿过平面 $x_1 = \text{常数}$ 的平均速度为 0，所以这些面上浓度的传递只与其在径向上的分布有关。在前述假设下，浓度径向变化很小，且可由下述方程得出：

$$\frac{\partial^2 C}{\partial z^2} + \frac{1}{z} \frac{\partial C}{\partial z} = \frac{a^2 v_0}{D} \left(\frac{1}{2} - z^2 \right) \frac{\partial C}{\partial x_1}$$

(方程的) 一个解为 $C = C_{x_1} + A \left(z^2 - \frac{1}{2} z^4 \right)$ ，其中 $C_{x_1} = C|_{z=0}$ 。

常数 A 满足 $A = \frac{a^2 v_0}{8D} \frac{\partial C_{x_1}}{\partial x_1}$

截面 x_1 处的转移速率为 (流量)

$$Q = +2\pi a^2 \int_0^1 v_0 \left(\frac{1}{2} - z^2 \right) C z dz$$

求得 $Q = -\frac{\pi a^4 v_0^2}{192D} \frac{\partial C_{x_1}}{\partial x_1}$

由假设，可将 C_{x_1} 替换为截面平均浓度 C_m ，即

$$Q = -\frac{\pi a^4 v_0^2}{192D} \frac{\partial C_m}{\partial x_1}$$

这相当于扩散系数为 $D_1 = \frac{a^2 v^2}{192D}$, 推导见下:

由连续性方程 (物质守恒)

$$\frac{\partial Q}{\partial x_1} = -\pi a^2 \frac{\partial C_m}{\partial t}$$

得到轴向扩散方程

$$D_1 \frac{\partial^2 C_m}{\partial x_1^2} = \frac{\partial C_m}{\partial t}$$

如果初始时, 溶质在 $x=0$ 处进入充有溶剂的管内, 则浓度分布满足

$$\frac{C}{C_0} = \begin{cases} \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left[x_1 \left(\frac{48D}{a^2 v^2 t}\right)^{\frac{1}{2}}\right] & (x_1 < 0) \\ \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[x_1 \left(\frac{48D}{a^2 v^2 t}\right)^{\frac{1}{2}}\right] & (x_1 > 0) \end{cases}$$

其中, $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-z^2} dz$

Chap. 3 Hydrostatics in porous media

3-1 principles of hydrostatics (静力学)

1° The statics of one fluid phase : ~~adsorption~~ ^{adsorption} at the surface of the pores (孔隙)

2° The statics of two phases of one fluid :
capillary condensation (毛细管凝结作用)

3° The statics of two different immiscible fluids (不混溶液体):

quasistatic displacement (准静态驱替)

4° wettability (润湿性)

3-2 Absorption of fluids by porous media

Theory of isotherms (等温线):

~~Adsorption~~ ^{Adsorption} / desorption isotherms (吸附/脱附作用等温线):

the relationship between fluid pressure and fluid density at a given temperature inside the porous medium, which is found experimentally.

1° Gibbs ~~adsorption~~ ^{adsorption} equation (吉布斯吸附作用方程式):

$$\omega = \frac{c}{RT} \frac{\partial \gamma}{\partial c}$$

where ω is the excess ~~number~~ ^(超额数) number of adsorbed molecules per unit area ;

T is the absolute temperature ;

γ is the surface tension.

2° Assume ^{that} adsorption could ~~only~~ occur only in a single layer and that the energy of adsorption would be the same everywhere on the adsorption surface. The relation between the quantity of gas adsorbed and the gas pressure is

$$\omega = \frac{abp}{1+ap}$$

where a and b are constants.

3° Assume adsorbed molecules exist in more than one layer, we have the BET equation:

$$\frac{\beta}{V(1-\beta)} = \frac{1}{V_m G} + \frac{\beta(G-1)}{V_m G}$$

where $\beta = \frac{P}{P_0}$ is the relative pressure (相对压力),
 P_0 is the vapour pressure of the bulk liquid
 (大块液体的蒸气压);

V_m is the volume of gas required to cover the entire internal surface of the porous medium.

V_m and G are constants.

4° Assume the condition ^{that} governs the correlation between the surface spreading force of adsorbed film and the area occupied per adsorbed molecule is the same with the adsorption of gases on solids. We have Harkins-Jura equation:

$$\lg \beta = B - \frac{A}{V^2}$$

where A and B are constants, and

$S = cA^{\frac{1}{2}}$, ~~where~~ S is the internal surface area, c is a constant.

Using BET equation, we can determine constant V_m , and if the ~~volume~~ volume required to cover a unit area of adsorbent (吸附剂) with a monomolecular layer of adsorbate (吸附质) V_0 is known, we can determine the internal area S :

$$S = \frac{V_m}{V_0}$$

If the media have too narrow interstices (裂缝), modification of the BET equation has to be used.

Harkins-Jura equation can be applied immediately to surface-measurements if the constant c can be evaluated.

3.3 Capillary condensation (毛细凝聚作用) of fluids in porous media

Theory of sorption hysteresis (吸附迟滞理论):

- Assume:
- i) all adsorption is entirely due to capillary condensation
 - ii) adsorbate densities equal bulk densities
 - iii) differences of pore shape from circular can be ignored
 - iv) the validity of the kelvin equation, including constancy of γ and ρ , is unimpaired at low values of a .

Then we have the kelvin equation.

A special case of kelvin equation, in case of zero contact angle and a circular capillary of radius a , is

$$\ln \frac{p_0}{p} = \frac{2\gamma M}{\rho a R T}$$

where p is the vapour pressure of a liquid, and p_0 is its original value.

M is the molecular weight (分子量).

T is the absolute temperature.

Sorption hysteresis is not yet completely understood.

Methods for determining the pore size distribution based on adsorption isotherms:

a) Methods based on capillary condensation theory

Assume γ and ρ are unaffected by a , then ^{the} kelvin equation suggests that the radius of the smallest pores that may be invaded by a given liquid at a constant temperature is solely a function of the pressure p .

From adsorption isotherms, we can get the pore size distribution.

b) Methods based on the determination of the internal surface.

For a straight circular cylinder, $a = \frac{2V}{S}$.

Assume the relation is approximately true to the pores of a porous medium, then we can get 'average' pore radii.

c) Methods yielding ~~approx~~ approximate estimates based on a consideration of the shape of adsorption isotherms.

3.4 The quasistatic displacement of one liquid by another in a porous medium

① Theory of capillary pressure

saturation (饱和度) $S = \frac{\text{space filled by the fluid}}{\text{pore space}}$

volumetric liquid content (液体的体积含量) $\theta = \frac{V_{\text{liquid}}}{V_{\text{bulk}}} = SP$

liquid ratio (液体比) $\phi = (1+\epsilon)\theta = \frac{SP}{1-P}$

~~Three~~

Three saturation regimes: (饱和状态)

a) Complete saturation regime (完全饱和状态)

b) Pendular regime (悬空状态):

The phase occurs in the form of pendular bodies throughout the porous medium. The pendular bodies do not touch each other so that there is no flow of that phase.

c) Funicular regime. (通道饱和状态):

The pendular bodies touch each other and merge.

There is a continuous network of both phases across the porous medium.

capillary pressure (毛细管压力) P_c : a pressure differential (差) formed by surface tension to create equilibrium.

○ $P_c = P_c(S) = P(\text{phase 1}) - P(\text{phase 2})$

The difference in free surface energy between two saturation states in the same porous medium is

○ $\Delta F = - \int_{S_A}^{S_B} P_c ds$

Since interfacial tension (界面张力) is by definition the free surface energy per unit interfacial area: $\gamma_{12} = \frac{dF}{d\Sigma}$,

we have
$$\Sigma_B - \Sigma_A = \frac{1}{\gamma_{12}} \int_{S_A}^{S_B} p_c ds$$

In a single capillary, the pressure differential across the interface is

○
$$p_c = \frac{2\gamma}{r}$$

where r is the curvature (曲率) of the interfacial

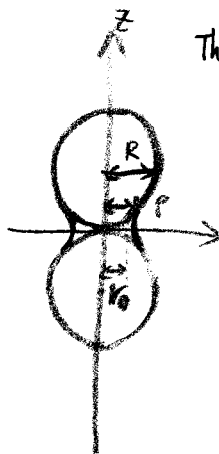
○ surface, $r = \frac{\delta_c}{2\cos\theta}$. θ is the contact angle.

Together we have,
$$p_c = \frac{4\gamma\cos\theta}{\delta_c}$$

If the displaced fluid is the vacuum, all the capillaries with a diameter larger than δ_c will be totally filled. Then we have

$$S = \int_{\delta = \frac{4\gamma\cos\theta}{p}}^{\infty} \alpha(\delta) d\delta$$

where we should replace $\frac{2}{r}$ by $\frac{1}{r_1} + \frac{1}{r_2}$, if the capillaries are not circular. r_1 and r_2 are the principle radii of curvature of the meniscus.



The shape of the meniscus satisfies

○
$$\frac{1}{r} \frac{d}{dr} \left(\frac{r \frac{dz}{dr}}{\sqrt{1 + (\frac{dz}{dr})^2}} \right) = \frac{p_c}{\gamma}$$

If $\frac{p_c}{\gamma} = 0$, the equation gives a catenoid (悬链曲面):

○
$$r = r_0 \cosh\left(\frac{z}{r_0}\right)$$

And $r_0 = 0.683R$

$p = 0.827R$

$V = 0.0521 \cdot \frac{4}{3}\pi R^3$, V is the volume of fluid in the pendular ring (液悬环).

For arbitrary porous media, the capillary pressure is

$$p_c = \frac{\gamma S}{P}$$

where S is the specific internal area (比内表面积).

If the porous medium is unconsolidated (不固结的), and S_0 is the particle surface per unit solid volume, then

$$S = (1-P)S_0$$

For particles of uniform diameter δ , $S_0 = \frac{6}{\delta}$, and

$$p_c = \frac{6(1-P)\gamma}{P\delta}$$

This may be used for arbitrary porous media.

① Leverett function: $J = \frac{p_c}{\gamma} \left(\frac{k}{P}\right)^{\frac{1}{2}} = J(S)$ (k is the permeability (渗透率) of the medium.)

Data for a number of unconsolidated porous media fall satisfactorily near the curve for imbibition of the wetting fluid (润湿相流体) (吸水) and the curve for drainage (排驱).

② Measurement of capillary pressure.

③ Capillary pressure and pore size distribution
mercury injection method.

④ Hydrostatic equilibrium of fluids in vertical columns of porous media
Fluid and air in a vertical porous medium will ~~reach~~ establish an equilibrium. The equilibrium condition is:

$$1^\circ -p_c(S) + \rho g y = \text{Const.}$$

where y is the vertical coordinate upward

$$\text{or } 2^\circ \Psi - z = \text{const.} = -z$$

where $\Psi = -\frac{p_c}{\rho g}$, z is the vertical coordinate measured downward,

z is the water depth over soil.

If the porous medium may swell by the addition of fluid, then the equilibrium condition becomes:

$$3^\circ \Psi - z + \Omega = \text{const.}$$

where $\Omega = \frac{de}{dz} p(z)$, is an overburden potential (过载位势).

$p(z)$ is the total vertical load carried by the porous media at z , expressed in centimeters of water (厘米水柱).

$$p(z) = p(z_0) + \int_{z_0}^z \frac{\rho + \frac{\rho_c}{P_w}}{1+e} dz$$

Profiles are called xeric (干燥的), if $\theta \rightarrow \theta_p$ as $z \nearrow$, and $\theta < \theta_p$; profiles are called hydric (含水的), if $\theta \rightarrow \theta_p$ as $z \nearrow$, and $\theta > \theta_p$. θ_p represents the pycnotatic point (转折点).

⑤ Limitations of the capillary pressure concept

The capillary pressure curves are different for drainage and imbibition experiments, which indicate hysteresis.

3.5 Relative Wettability (相对润湿性)

① The concept of wettability

Ordinal scales of wettability for fluids in relation to a
(数位标示)

given porous medium are denised.

② Measurement of wettability

a) The measurement of ~~displacement~~ displacement pressure (驱替压力)

b) Imbibition tests (吸水测试)

c) Fractional wettability (润湿比率) determinations.

③ Heat of wetting (润湿热)

$$W = S_1 \gamma_{SA} - S_1 \gamma_{SL} - S_2 \gamma_{LA}$$

where, W is the heat of wetting; S_1 is the solid-liquid interface area, and S_2 the liquid-air interface area.

With $\cos \theta = \frac{\gamma_{SA} - \gamma_{SL}}{\gamma_{LA}}$, we have

$$W = \gamma_{LA} (S_1 \cos \theta - S_2)$$

If $S_1 \gg S_2$ and $S_1 = S$, the total internal area, we have

$$W = \gamma_{LA} S \cos \theta$$

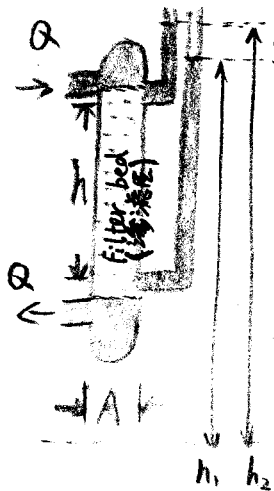
Chap. 4 DARCY'S LAW

4.1 The Flow of Homogeneous Fluids (均质流体) in Porous Media

4.2 Experimental Investigations

① Darcy's experiment

$$1^{\circ} Q = -KA \frac{h_2 - h_1}{h} \quad (\text{Darcy's law})$$



where \$Q\$ is volume flow rate;
 \$A\$ is the horizontal plane area;
 \$h, h_1, h_2\$ are heights denoted on the graph.
 \$k\$ is a constant.

$$2^{\circ} Q = -k'A \frac{p_2 - p_1 + \rho gh}{h}$$

where \$k'\$ is a constant, \$k' = \frac{k}{\rho g}\$.
 \$p_1, p_2\$ are the pressure at the upper and lower boundary of the bed.

② The permeability concept

Specific Permeability (比渗透率): $k = \frac{K}{\mu}$

where \$\mu\$ is the viscosity of the fluid.

The dependence of the permeability on the fluid pressure \$p\$ and the total pressure \$J\$ is

$$k = k(J - p)^{-m}$$

where \$m\$ is a constant.

A directional variation of permeability may occur. That is, if a cube is cut out of a macroscopically homogeneous piece of rock, the permeability may not be the same across all the faces.

4.3 Differential Forms of Darcy's Law

① Isotropic porous media

$$\vec{q} = -\frac{k}{\mu} (\nabla p - \rho \vec{g})$$

where \vec{q} is called the local filter-velocity (局部渗透速度).

Note: 1° The differential equation can be written in a form of force potential (力势):

$$\vec{q} = -\frac{k\rho}{\mu} \nabla \phi$$

$$\text{where } \phi = gz + \int_P \frac{dP}{\rho}$$

2° Another form of Darcy's law is

$$\vec{q} = -\nabla \left(\frac{k\rho}{\mu} \right) + \frac{k\rho \vec{g}}{\mu}$$

$$\text{or, } \vec{q} = -\nabla \psi$$

where $\psi = \frac{k\rho}{\mu} + \int_z \frac{k\rho g}{\mu} dz$ is a velocity potential (速度势).

However, this differential representation is wrong

For given boundary conditions, the differential form of Darcy's law is by itself not sufficient, for it contains (\vec{q}, p, ρ) . Two further equations are needed:

i) Connection between ρ and p of the fluid

$$\rho = \rho(p)$$

ii) Continuity condition

$$-\rho \frac{\partial p}{\partial t} = \nabla \cdot (\rho \vec{q})$$

where P is the porosity (孔隙度)

All the three equations lead to:

$$P \frac{\partial p}{\partial t} = \nabla \cdot \left[\frac{k\rho}{\mu} (\nabla p - \rho \vec{g}) \right]$$

② Anisotropic (各向异性) porous media

$$\vec{q} = - \frac{\bar{k}}{\mu} \rho \cdot \nabla \phi$$

where \bar{k} is the permeability tensor of the porous medium, and it's symmetric. ϕ is defined as before.

1° If we define directional ~~permeability~~ permeability ^(定向渗透率) as

$$k'_n = \frac{\mu q_n}{\phi_n \rho}$$

$$\text{where } \phi_n = \vec{n} \cdot \nabla \phi = \frac{\mu}{\rho} \vec{n} \cdot \bar{k}^{-1} \cdot \vec{n} q_n,$$

$$\text{then } k'_n = \frac{1}{\vec{n} \cdot \bar{k}^{-1} \cdot \vec{n}}$$

$$\text{and } \frac{1}{k'_n} = \frac{\cos^2 \alpha}{k_1} + \frac{\cos^2 \beta}{k_2} + \frac{\cos^2 \gamma}{k_3},$$

This is an ellipsoid if $r = \sqrt{k'_n}$.

2° If we define directional permeability as

$$k''_n = \vec{n} \cdot \bar{k} \cdot \vec{n}$$

$$\text{then } k''_n = k_1 \cos^2 \alpha + k_2 \cos^2 \beta + k_3 \cos^2 \gamma.$$

This is an ellipsoid if $r = \frac{1}{\sqrt{k''_n}}$.

In most cases, the difference between the two types of directional permeability is quite negligible.

③ Compressible porous media.

1° Assume: a) Both fluid and medium follow Hooke's law:

$$\begin{cases} \frac{dP}{dP} = \beta_f \\ \frac{dP}{dP} = \beta_m \end{cases}$$

where β_f and β_m are compressibility coefficients, assumed to be constants.

b) The permeability of the medium does not change during compression.

Then the compressibility of the medium only alters the continuity equation.

We have : (for proof, see pp. 84)

$$\nabla \cdot (\rho \vec{q}) = - \left(p + \frac{\beta_m}{\beta_f} \right) \frac{\partial p}{\partial t}$$

This (Shchelkachev's) theory is incomplete because it doesn't assume that permeability depends on the prevailing fluid pressure.

2° For compressible porous media, we have :

Darcy's law : $q_\alpha = - \frac{k_{\alpha\beta}}{\mu} \frac{\partial p}{\partial \xi_\beta}$
 where the subscripts denote components.

Constitutive equation: $\rho = \rho(p)$

Continuity condition: $-\frac{\partial(\rho q_\alpha)}{\partial \xi_\alpha} = \frac{\partial[(1+K)\rho p]}{\partial t}$

where $(K+1)$ is the volume factor induced by the consolidation of the porous medium.

The three equations lead to an fundamental hydrodynamic equation: (基本动力学方程)

$$\frac{\partial[(1+K)\rho p]}{\partial t} = \frac{\partial \left(\frac{\rho}{\mu} k_{\alpha\beta} \frac{\partial p}{\partial \xi_\beta} \right)}{\partial \xi_\alpha}$$

If we assume that the total change of bulk volume (总体积) is entirely produced by a change in the pore volume, then the fundamental hydrodynamic equation reduces to :

$$\frac{\partial}{\partial t} [\rho (p_0 + K(p, \mathcal{J}))] = \frac{\partial}{\partial \xi_\alpha} \left(\frac{\rho}{\mu} k_{\alpha\beta} \frac{\partial p}{\partial \xi_\beta} \right)$$

3° In the case of locally isotropic stress and permeability.

Permeability : A unit volume is under pressure \mathcal{J} and p ,
 the relative volume reduction is $-K(\mathcal{J}, p)$.

Superficial permeability (宏观渗透率) satisfies :

$$\frac{Q}{A_0} = \frac{k_{sup}}{\mu} \frac{p_2 - p_1}{L_0}$$

where the index 0 indicate the original value.
 (zero strain state)

The true permeability satisfies:

$$\frac{Q}{A_0(1+\frac{2}{3}K)} = \frac{k_{true}}{\mu} \frac{P_2 - P_1}{L_0(1+\frac{K}{3})}$$

$$\text{Then } k_{true} = \frac{1+\frac{K}{3}}{1+\frac{2}{3}K} k_{sup} = (1-\frac{K}{3}) k_{sup}.$$

Here we assumed the volume change in the pressure cell (压力室) is cubic (三方向均匀的).

Assume the true permeability k_{true} under pressure is the same in the field ^(实地) as in the pressure cell.

Since the compaction (压实作用) in the field occurs in the vertical direction ξ only, the superficial permeabilities in direction ξ, η, ζ satisfy:

$$\frac{Q_x}{L_0^2(1+K)} = \frac{k_{true}}{\mu} \frac{P_2 - P_1}{L_0}$$

$$\frac{Q_y}{L_0^2(1+K)} = \frac{k_{true}}{\mu} \frac{P_2 - P_1}{L_0}$$

$$\frac{Q_z}{L_0^2} = \frac{k_{true}}{\mu} \frac{P_2 - P_1}{L_0(1+K)}$$

that is,

$$k_{\xi\xi} = (1+K) k_{true} = (1+\frac{2}{3}K) k_{sup}$$

$$k_{\eta\eta} = k_{\xi\xi} = (1+\frac{2}{3}K) k_{sup}$$

$$k_{\zeta\zeta} = \frac{1}{1+K} k_{true} = (1-\frac{4}{3}K) k_{sup}$$

The permeability tensor in (ξ, η, ζ) has the form:

$$K_{\alpha\beta} = k_{sup} \begin{pmatrix} 1+\frac{2}{3}K & & \\ & 1+\frac{2}{3}K & \\ & & 1-\frac{4}{3}K \end{pmatrix}$$

Porosity: Measure $\frac{1}{P_0} \frac{d}{dP} [(1+K)P]$, where the index 0 refers to the zero strain state.

If the rheological equation (流变方程) of the fluid $f(P)$ is known, then by integration, we can obtain

$$(1+K)P.$$

note: (ξ, η, ζ) are zero strain coordinates (零应变坐标) of the field.

4° In the case of anisotropic stresses ~~and~~ ^{and} permeability.

Assume that the vertical direction is a principal direction for stress and for permeability.

~~Permeability~~

The total stresses T_{zz} , T_{xx} , T_{yy} should be set in such a manner that T_{zz} should be the overburden pressure in the field and there is no lateral strain (横向应变) of the rock.

Permeability: Measure k_x, k_y, k_z separately for all values of the fluid pressure p . This yields the permeability tensor $k_{\alpha\beta}$:

$$k_{\alpha\beta} = \begin{pmatrix} k_x & & \\ & k_y & \\ & & k_z \end{pmatrix}$$

Here the parameters k_{α} refer to the ~~field~~ field state. (实地状态).

Porosity: It can be measured just as in the isotropic case. Now, P_0 refers to the porosity of field state.

5° For plastic porous media with hysteresis.

Denote $\theta = \frac{1}{3} \sigma_{ij}^F$

$$\beta_p = \left(\frac{\partial P}{\partial p} \right)_\theta$$

$$\beta'_\theta = - \left(\frac{\partial P}{\partial \theta} \right)_{\text{loading}}$$

$$\beta''_\theta = - \left(\frac{\partial P}{\partial \theta} \right)_{\text{unloading}}$$

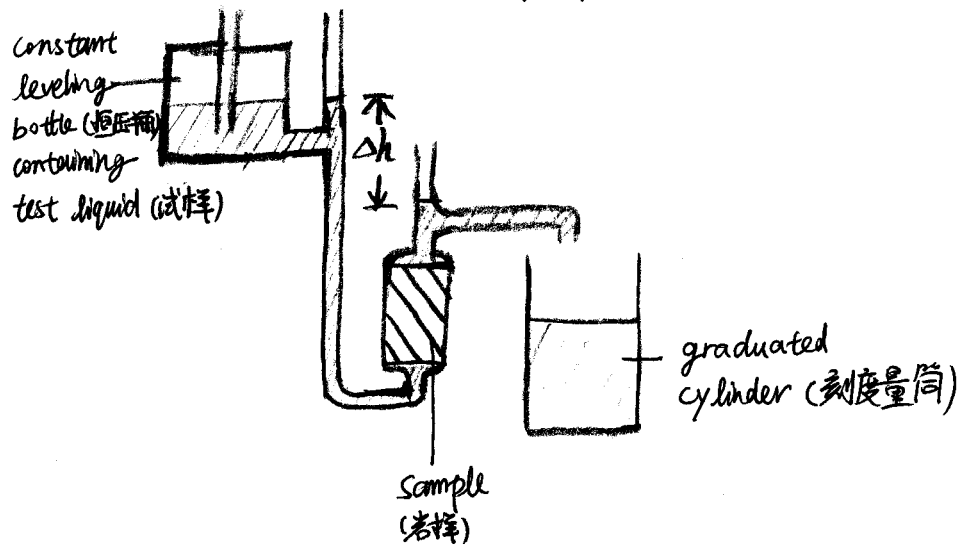
The Barenblatt - Krjlov equations are:

$$\frac{\partial P}{\partial t} = a_1^2 \nabla^2 P, \quad a_1^2 = \frac{k(P_0)}{\mu [P_0 \beta_f + \beta_p + \beta'_\theta]} \quad \left(\frac{dp}{dt} \leq 0 \right)$$

$$a_2^2 \nabla^2 P, \quad a_2^2 = \frac{k(P_0)}{\mu [P_0 \beta_f + \beta_p + \beta''_\theta]} \quad \left(\frac{dp}{dt} > 0 \right)$$

4.4 The Measurement of Permeability

① Laboratory methods for isotropic permeability determinations



When measuring permeability in highly compactible substances such as soils and clays, one should pay special precautions to the pore pressure.

② Anisotropic permeability ~~measurements~~ measurements.

In principle, there are two ways to measure anisotropic permeability, as are discussed in 4.3.2. One can use either of them indiscriminately, because their results differ only slightly.

In vast majority of cases, the horizontal permeability in underground rock strata is much greater than the vertical permeability.

③ In situ (就地) permeability measurements.

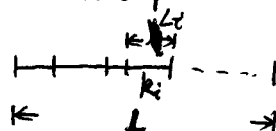
It is desirable to measure the permeability in situ when it's difficult to obtain a representative sample of the medium in question.

④ Averaging of permeability measurements.

In most instances, a natural porous medium is inhomogeneous on a small scale, but it can be considered as homogeneous on a large scale.

In practice, what is wanted is generally the 'effective' permeability of the medium on a large scale. However, chances are that measurements can only be performed on small scale sample. Now, we need to determine the 'effective' permeability from 'local' permeabilities. ^{these}

1° Extreme case 1: constant local flux, or a group of horizontal porous tubes in sequence.



Darcy's law states that $q_i = \frac{k_i}{\mu} \frac{P_i - P_{i-1}}{L_i}$, ($i=1, \dots, n$)

Then we have $\mu q \sum_{i=1}^n \frac{L_i}{k_i} = P_n - P_0$.

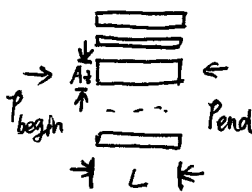
Define 'effective' permeability k_{eff} :

$$q = \frac{k_{eff}}{\mu} \frac{P_n - P_0}{L}$$

then $\frac{1}{k_{eff}} = \sum_{i=1}^n \frac{L_i}{L} \frac{1}{k_i}$, where $\frac{L_i}{L}$ can be interpreted as the relative frequency f_i (频率) at which the permeability k_i was found in small-sample measurements.

This is the harmonic mean of local permeability measurements.

2° Extreme case 2: constant pressure gradient, or a group of horizontal porous tubes in parallel.



Darcy's law gives $Q_i = A_i q_i = A_i \frac{k_i}{\mu} \frac{P_{end} - P_{begin}}{L}$

Then we have $q = \frac{Q}{A} = \frac{1}{\mu} \left(\sum \frac{A_i}{A} k_i \right) \frac{P_{end} - P_{begin}}{L}$

The effective permeability of the system is:

$$k_{eff} = \sum \frac{A_i}{A} k_i$$

where $\frac{A_i}{A}$ can be interpreted as the relative frequency f_i , at which the permeability k_i was found in small samples.

This is the arithmetic mean of local permeability measurements.

The effective permeability ^{lies} indeed somewhere between the arithmetic and harmonic means.

Empirically, the practice has evolved of employing the geometric mean for averaging:

$$\lg k_{\text{eff}} = \sum f_i \lg k_i$$

However, it is correct only in very rare cases.

The statistics of the spatial distribution of the local permeabilities is ~~not~~ necessary in further discussion of averaging.

4.5 Filtration Theory (渗滤理论)

① Build-up (累积) of a filter cake (滤饼)

A slurry (悬浮液) is filtered through a filter cake, the rate of deposition (沉积) is proportional to the throughput (流量) of filtrate (滤液):

$$\frac{dh}{dt} = cq$$

Darcy's law reads:

$$q = \frac{k}{\mu} \frac{P_2 - P_1}{h} \quad (\text{constant pressure})$$

Then we have

$$cq = -\frac{k}{\mu} \frac{P_2 - P_1}{q^2} \frac{dq}{dt}$$

the solution is

$$q = \sqrt{\frac{k(P_2 - P_1)}{2\mu(ct + c')}}}$$

The total volume of filtrate filtered through a unit area from the beginning of the process of filtration is

$$V(t) = \int_{t_0}^t q dt = \frac{1}{c} \sqrt{\frac{2k}{\mu} (P_2 - P_1) (ct + c')} + V_0$$

② Colmatage (堵塞)

Colmatage refers to the trapping of particles contained in a stream of fluid during its passage through a porous medium.

Assume ~~the~~ $\frac{\partial k}{\partial t}$ is proportional to the concentration of still unblocked pores, where k is the concentration of the pores of the porous medium blocked:

$$\frac{\partial k(x,t)}{\partial t} = a C(x,t) [A - k(x,t)]$$

where A is the initial number of ^{unblocked} pores per unit volume.

$C(x,t)$ is the concentration of particles in the fluid stream.

The mass balance equation is

$$\frac{\partial C(x,t)}{\partial t} + \frac{\partial}{\partial x} (CV) = -\frac{\partial k(x,t)}{\partial t}$$

The two equations describe the colmatage process.

Chap. 5 SOLUTIONS OF DARCY'S LAW

5-1 General Remarks

5-2 Steady State Flow

① Analytical solutions

The fundamental hydrodynamic equation for isotropic porous media is

$$\rho \frac{\partial p}{\partial t} = \nabla \cdot \left[\frac{\rho k}{\mu} (\nabla p - \rho \vec{g}) \right]$$

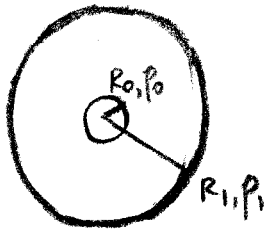
In steady state flow, $\frac{\partial}{\partial t} = 0$. This gives

$$\nabla \cdot \left[\frac{\rho k}{\mu} (\nabla p - \rho \vec{g}) \right] = 0$$

If the fluid is incompressible and the porous medium homogeneous, we have

$$\nabla^2 p = 0$$

For two-dimensional radial flow of an incompressible fluid into a well which is completely penetrating the fluid-bearing medium, the solution is



$$Q = \frac{2\pi k}{\mu \ln \frac{r_1}{r_0}} (p_1 - p_0)$$

where r_0 is the radius of the well, p_0 is the pressure at its surface, p_1 is the pressure at distance r_1 from the well, Q is the total discharge (总产量) per unit time and unit penetration length of the well.

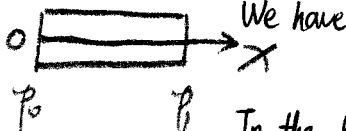
If the fluid is compressible, let

$$\chi = \int_{p_0}^p \rho dp$$

the equation reads

$$\nabla^2 \chi - \nabla \cdot (\rho^2 \vec{g}) = 0$$

For gases and horizontal flow of liquids, the second term is negligible.

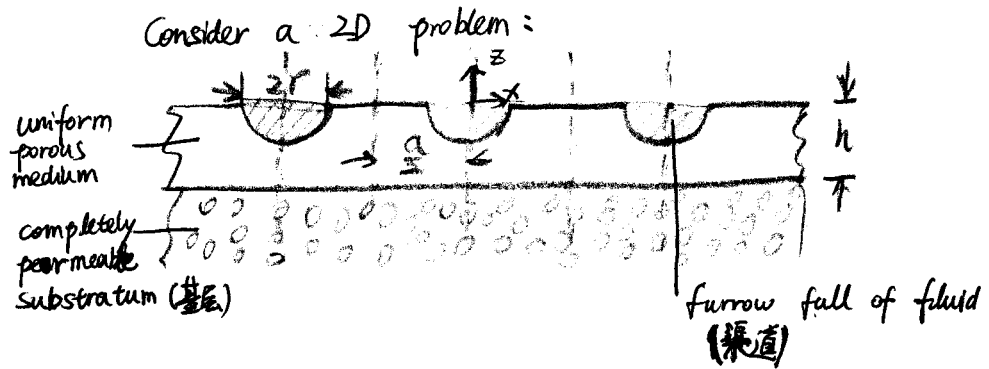


$$\nabla^2 \chi = 0$$

In the linear case for an ideal gas, $\chi = c' + bx$

with boundary conditions, the solution is

$$p^2 = p_0^2 + \frac{\chi}{L} (p_L^2 - p_0^2)$$



The equation reads:

$$\nabla^2 \psi = 0$$

where $\psi = \frac{k}{\mu} (p + \rho g z)$ is the velocity potential.

Using the method of images, the solution is

$$\psi = A \sum_{m=-\infty}^{m=\infty} \ln \left(\frac{\cosh \left[\frac{\pi(x-ma)}{2h} \right] - \cos \left(\frac{\pi z}{2h} \right)}{\cosh \left[\frac{\pi(x-ma)}{2h} \right] + \cos \left(\frac{\pi z}{2h} \right)} \right) + \psi_0$$

where ψ_0 is the velocity potential at the top of the substratum, A is a constant.

The stream function corresponding to the velocity potential is

$$\varphi = 2A \sum_{m=-\infty}^{m=\infty} \arctan \left(\frac{\sinh \left(\frac{\pi(x-ma)}{2h} \right)}{\sin \left(\frac{\pi z}{2h} \right)} \right) + \varphi_0$$

② Solutions by analogies (模拟)

1° The flow of electricity

2° The flow of heat

3° The distribution of stresses

4° The flow of viscous fluids

5° Mechanical scaling (力学物理标度法)

5.3 Unsteady State Flow

① General remarks

② Theory of linear approximation of unsteady state flow
 1° If the fluid is a liquid of constant compressibility.

$$\rho = \rho_0 e^{\beta_f (P - P_0)}$$

Neglect the gravity term in

$$\rho \frac{\partial \rho}{\partial t} = \nabla \cdot \left[\frac{\rho k}{\mu} (\nabla P - \rho \vec{g}) \right]$$

We get

$$\rho \beta_f \frac{\partial \rho}{\partial t} = \frac{k}{\mu} \nabla^2 \rho$$

2° If the porous medium is elastic.

The continuity equation reads:

$$\nabla \cdot (\rho \vec{q}) = - \left(\rho + \frac{\beta_m}{\beta_f} \right) \frac{\partial \rho}{\partial t}$$

Neglect the gravity term in Darcy's law, we get

$$\nabla \cdot \left(\rho + \frac{\beta_m}{\beta_f} \right) \frac{\partial \rho}{\partial t} = \nabla \cdot \left(\frac{\rho k}{\mu} \nabla P \right)$$

With $\frac{d\rho}{dP} = \rho \beta_f$, we have

$$\nabla P = \frac{1}{\rho \beta_f} \nabla \rho$$

We get

$$\left(\rho \beta_f + \beta_m \right) \frac{\partial \rho}{\partial t} = \frac{k}{\mu} \nabla^2 \rho$$

Note: If we suppose $\rho \approx \rho_0 + \beta_f \rho_0 (P - P_0)$,

$$\text{then } \left(\rho \beta_f + \beta_m \right) \frac{\partial \rho}{\partial t} = \frac{k}{\mu} \nabla^2 \rho$$

3° If compressibilities are small.

The continuity equation reads: (Werner)

$$\nabla \cdot \vec{q} = - \bar{\rho} \left(\rho \beta_f + \beta_m \right) \frac{\partial \phi}{\partial t}$$

where $\bar{\rho}$ is the average fluid density,

ϕ is the force potential.

With Darcy's law $\vec{q} = - \frac{k \rho}{\mu} \nabla \phi$, we get

$$\left(\rho \beta_f + \beta_m \right) \frac{\partial \phi}{\partial t} = \frac{k}{\mu} \nabla^2 \phi$$

4° For an ideal gas.

$$P = c\rho$$

Neglect the gravity term, we get

$$2P \frac{\partial P}{\partial t} = \frac{k}{\mu} \nabla^2 (P^2)$$

Under certain conditions, we have

$$\frac{\partial (P^2)}{\partial t} = \text{Const.} \nabla^2 (P^2)$$

For the four linearized cases, it is always the heat-conductivity equation.

③ Various specific solutions of the linearized equations

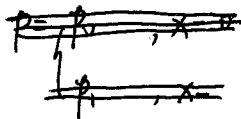
1° For a compressible porous medium in linear arrangement, the fundamental equation reads

$$a^2 \frac{\partial^2 P}{\partial x^2} = \frac{\partial P}{\partial t}$$

$$\text{where } a^2 = \frac{k}{\mu(P\beta_f + \beta_m)}$$

The boundary condition is

initial



$$\begin{cases} P(0, t) = P_0 \\ P(L, t) = P_1 \\ P(x, 0) = P_0 \end{cases}$$

The solution is

$$P(x, t) = P_0 - (P_0 - P_1) \frac{x}{L} + \frac{2}{\pi} (P_0 - P_1) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \exp\left(-\frac{n^2 \pi^2 a^2}{L^2} t\right) \sin\left(\frac{n\pi x}{L}\right)$$

If denote

$$\varepsilon = \frac{x}{L}, \quad F = \frac{2a^2 t}{L^2}, \quad W = \frac{P_0 - P}{\varepsilon(P_0 - P_1)}$$

Then the solution becomes

$$W = 1 - \frac{2}{\varepsilon \pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \exp\left(-\frac{n^2 \pi^2 F}{2}\right) \sin(n\pi \varepsilon)$$

2° In polar coordinates, the equation becomes

~~$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{1}{a^2} \frac{\partial p}{\partial t}$$~~

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{1}{a^2} \frac{\partial p}{\partial t}$$

The initial and boundary conditions are

$$\begin{cases} p(r, 0) = p_0 \\ p(\infty, t) = p_0 \\ 2\pi \lim_{r \rightarrow \infty} (r \frac{\partial p}{\partial r}) = \frac{\mu}{k} Q \end{cases}$$

The solution is

$$\begin{aligned} p &= p_0 - \frac{\mu Q}{k 4\pi} \int_{\frac{r^2}{4a^2 t}}^{\infty} \frac{e^{-x}}{x} dx \\ &= p_0 + \frac{\mu Q}{k 4\pi} \cdot \text{Ei}\left(-\frac{r^2}{4a^2 t}\right) \end{aligned}$$

④ General unsteady-state flow

For an ideal gas, the equation is

$$\frac{\partial^2 \left(\frac{p}{p_0}\right)^2}{\partial x^2} = \frac{2p\mu}{p_0 k} \frac{\partial}{\partial t} \left(\frac{p}{p_0}\right)$$

If we set $\Delta t = \frac{1}{4} a (\Delta x)^2$, where $a = \frac{2p\mu}{p_0 k}$,
the difference scheme is

$$p_{x,t+\Delta t} = \frac{1}{4} (p_{x+\Delta x,t}^2 - 2p_{x,t}^2 + p_{x-\Delta x,t}^2) + p_{x,t}$$

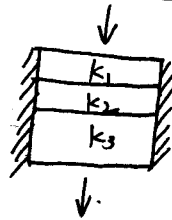
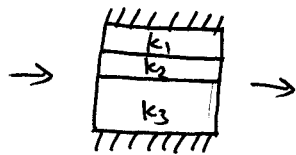
With certain initial and boundary conditions, we can solve the problem numerically.

⑤ Experimental solutions

Flow in channels and fractures analogies to Darcy's Law

channels and fractures can add significantly to flow capacity.

美比/5 透膜图



边条件: 等压、无流动

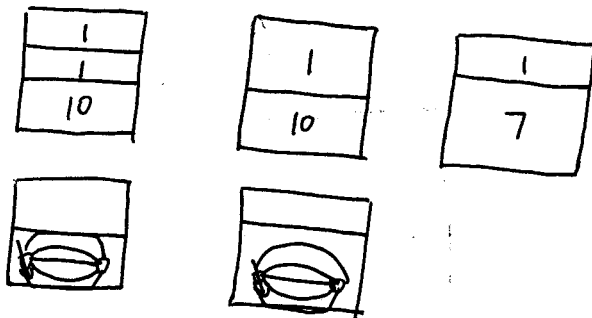
粗化的实质: 减少信息量, 保留关键性质。

粗化中保持的结果: 1° 产/注量

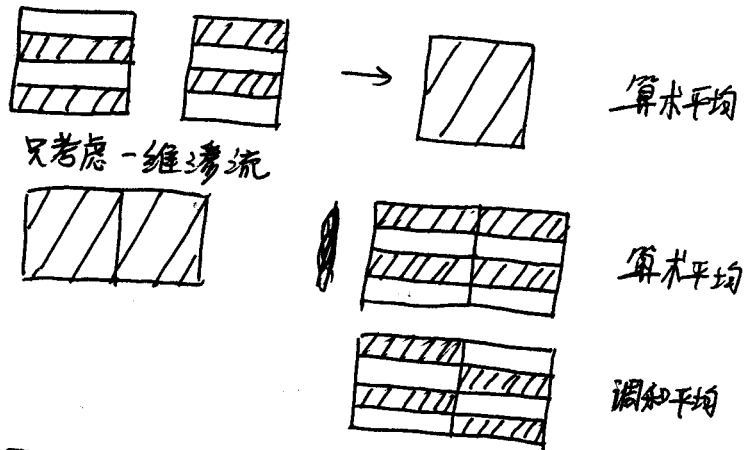
2° 压力分布 / 饱和度分布 (缝阻流)

3° 注入剂的突破时间 (增产措施; 多相流)

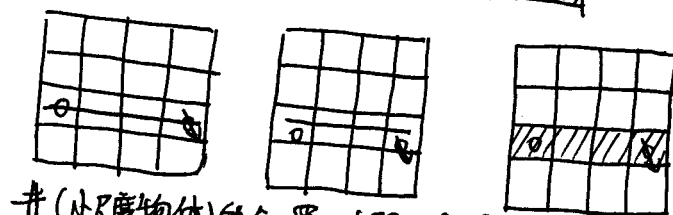
Case 1°:



Case 2°:



Case 3°:



井(小尺度物体)的位置对强各向异性的地层中的流动很敏感。

Upscaling

• existing upscaling techniques

single-phase upscaling — flow ~~Q, DP~~ (Q, DP)
local & global techniques ($k \rightarrow k^*$, $Trans^*$)

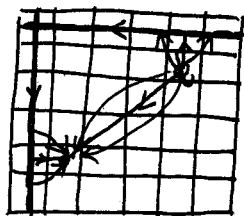
multi-phase upscaling — transport (oil cut)
pseudo relative perm. ($K_r \rightarrow K_r^*$)

"multi-scale" modeling

upscaling of flow (pressure equation)

fine scale solution of transport (saturation equation)

Case 4°



$$\frac{\Delta p}{\Delta L} = \frac{\mu q}{kA}$$

等流量压降梯度

$$L_{block} = 100m$$

$$L_{fracture} = 0.1mm$$

$$k_{fracture} \approx 5.4 \times 10^{10} \times \left(\frac{1}{2.54}\right)^2 \times \left(\frac{1}{10}\right)^2 md$$

$$\approx 10^8 md$$

$$k_{matrix} = 1000 md$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{fracture} : \left(\frac{\Delta p}{\Delta L}\right)_{matrix} = 10^5 = 1$$

均质模型忽略了强各向异性模型中的通道，流线会作错。

Case 5°: 粗化效果对边条件敏感

