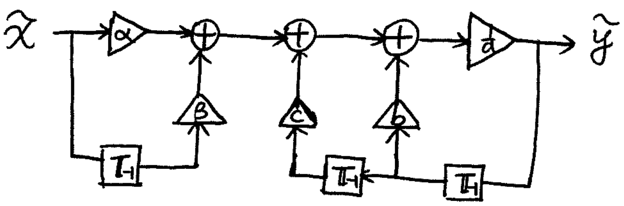
System diagram

$\boxed{G}$: device

$\triangleright_a$: multiplier

$\oplus$: adder

e.g.:



The diagram represents:

$$\tilde{y}(t) = \frac{1}{a}\{\alpha x(t)+\beta x(t-1) - [b y(t-1) + c y(t-2)]\}$$

where $\boxed{T_{-1}}$ is a unit-delay device.

Note: 1° $T_k = (T_1)^k$ $(k\in\mathbb{Z})$

so $T_{-1} = (T_1)^{-1}$.

Z-transform

1° pairs

$G_z(z)$	$g(t)$	
$\frac{1}{1-az^{-1}}$	$a^t U_r(t)$	$(\lvert a\rvert<1)$
$\frac{1}{1-az}$	$a^{-t}U_r(-t)$	$(\lvert a\rvert<1)$
$z^{-k}G_z(z)$	$g(t-k)$	

2° definition

$$Z\{x(t)\} \equiv \sum_{t=-\infty}^{+\infty} x(t) z^{-t} \qquad (z=e^{i2\pi f})$$

$$x(t) = \frac{1}{2\pi i}\oint_C X(z) z^{t-1}\, dz$$

$$= \mathrm{Res}_{\{v_d \text{ inside } C\}} (v_d, X(z)z^{t-1})$$

Region of convergence (ROC): the set of points z, where the limit in the definition of z-transform converges.

Identity

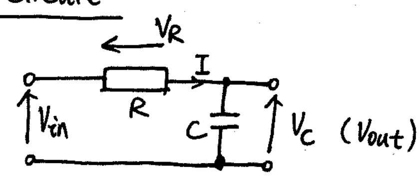
1° $\frac{1}{|c|}\sum_{k=0}^{c-1} e^{-i2\pi \frac{k}{c} t} = \sum_{r=-\infty}^{+\infty} \delta_k(t-rc)$

(or $I(t\%c=0)$)

2° zero-mean Gaussian r.v.'s.

$E(X_1X_2X_3X_4) = E(X_1X_2)E(X_3X_4) + E(X_1X_3)E(X_2X_4) + E(X_1X_4)E(X_2X_3)$

LC circuit



System function (frequency response):

$$H(f) = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+sRC} = \frac{1}{1+i2\pi f RC}$$

$(s = i2\pi f)$

Core Concepts in R.P.

Topic	Symbol	Focus
Convergence	m.s.s.	stable
random sequence/ waveform	$X(u,t)$	w.s.s. (psd.)
system	G	LTI
simulation	$X = GW$	causal
estimation	$\hat{Z} = GX + m$	

Shot Noise

$S(u,t) \qquad \mathcal{T} = \mathbb{R}$

$$\begin{cases} m_s = \lambda \\ R_s(\tau) = \lambda^2 + \lambda \delta_D(\tau) \\ S_s(f) = \lambda^2 \delta_D(f) + \lambda \end{cases}$$

not 2nd order process.

Ergo-dic

1° $\langle X(u,t) \rangle = \mathbb{E}X(u,t)$, $\forall u \in \mathcal{U}, \forall t \in \mathbb{R}$

2° $\mathbb{E}X(u,t) = m_X$,

$$\lim_{T\to\infty} \mathbb{E}\left| \frac{1}{2T}\int_{-T}^{T} X(u,t)\,dt - m_X \right|^2 = 0$$

3° $X(u,t)$ w.s.s. , $\lim_{\tau\to\infty} k_X(\tau) = 0$.