Ⅲ Examples of sufficient / complete statistics & UMVU estimators

	sufficient	complete	UMVU
exponential family ($\underline{\theta}$)	$(t_1(\underline{X}), \cdots, t_k(\underline{X}))$		
exponential family, iid ($\underline{\theta}$)	$\left(\sum_{j=1}^n t_1(X_j), \cdots, \sum_{j=1}^n t_k(X_j)\right)$	$\left(\sum_{i=1}^n t_1(X_i), \cdots, \sum_{i=1}^n t_k(X_i)\right)^*$	
$U(0,\theta)$	$X_{(n)}$	$X_{(n)}$	$\frac{n+1}{n} X_{(n)}$ for θ
Gaussian (μ, σ^2)	$\left(\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2\right)$, or $(\bar{X}, S^2)$	$\left(\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2\right)$ or $(\bar{X}, S^2)$	1° $(\bar{X}, S^2)$ for (μ, σ^2) 2° $\bar{X}^2 - \frac{S^2}{n}$ for μ^2
Bernoulli (θ)	$\sum_{i=1}^n X_i$	$\sum_{i=1}^n X_i$	1° $\bar{X}$ for θ 2° $\frac{(\sum_{i=1}^n X_i)(n - \sum_{i=1}^n X_i)}{n(n-1)}$ for $\theta(1-\theta)$
Poisson (θ)	$\sum_{i=1}^n X_i$	$\sum_{i=1}^n X_i$	
Gaussian (θ, θ^2)		not $\sum_{i=1}^n X_i$	
Gaussian (μ, b^2)	$\sum_{i=1}^n X_i$	$\sum_{i=1}^n X_i$	$\bar{X}$ for μ
Gaussian (a, σ^2)	$\sum_{i=1}^n X_i^2$	$\sum_{i=1}^n X_i^2$	$\hat{\sigma}^2$ for σ^2

* Note: If $\underbrace{\{(w_1(\underline{\theta}), \cdots, w_k(\underline{\theta})) : \theta \in \Theta\}}_{\text{the natural parameter space}}$ contains an open set in $\mathbb{R}^k$.

(Continued)

Exponential (λ)	$\sum_{i=1}^n X_i$	$\sum_{i=1}^n X_i$	$\left(1 - \frac{t}{\sum_{i=1}^n X_i}\right)^{n-1}$ for $g(\lambda) = e^{-\lambda t}$