

Cardan's formula

$$\begin{cases} I_1 = \text{tr}(A) \\ I_2 = \frac{1}{2}(I_1^2 - a_{ij}a_{ji}) \\ I_3 = \det(A) \end{cases}$$

$$\begin{cases} p = \frac{I_1^2 - 3I_2}{9} \\ r = \sqrt{p} \\ q = \frac{2I_1^3 - 9I_1I_2 + 27I_3}{54} \end{cases}$$

$$\theta = \cos^{-1}\left(\frac{q}{p^{\frac{3}{2}}}\right)$$

$$\begin{cases} \xi_1 = 2r \cos \frac{\theta}{3} \\ \xi_2 = 2r \cos\left(\frac{\theta}{3} + \frac{2\pi}{3}\right) \\ \xi_3 = 2r \cos\left(\frac{\theta}{3} + \frac{4\pi}{3}\right) \end{cases}$$

$$\begin{cases} \lambda_1 = \xi_1 + \frac{I_1}{3} \\ \lambda_2 = \xi_2 + \frac{I_1}{3} \\ \lambda_3 = \xi_3 + \frac{I_1}{3} \end{cases}$$

S-strain :

$$\begin{aligned} \vec{u}(\vec{r} + d\vec{r}) &= \vec{u}(\vec{r}) + \vec{\omega} \cdot d\vec{r} + \vec{\Gamma} \cdot d\vec{r} \\ &= \vec{u}(\vec{r}) + \vec{\omega} \times d\vec{r} + \vec{\Gamma} \cdot d\vec{r} \end{aligned}$$

$$\begin{cases} \vec{\omega} = \frac{1}{2}(\vec{u} \nabla - \nabla \vec{u}) \\ \vec{\Gamma} = \frac{1}{2}(\nabla \vec{u} + \vec{u} \nabla) \\ \vec{\omega} = \frac{1}{2}(\nabla \times \vec{u}) \end{cases}$$

$$\epsilon_{ij} = \frac{1}{2}(u_{ij} + u_{ji})$$

$$\epsilon = \vec{\xi} \cdot \vec{\Gamma} \cdot \vec{\xi}$$

$$\epsilon = \vec{\xi} \cdot \vec{\Gamma} \cdot \vec{\eta}$$

Thermal heat will only affect elongation, not shear.

$$\epsilon_{ij} = \alpha (T - T_0) \delta_{ij}$$

Volume dilation :

$$\Delta \equiv \epsilon_{ii} = I_1$$