Continuity and Differentiability in the mean-square sense. (Chap 13.2.2)

(1) Def: (Continuity in the mean-square sense)

r.p. $X(u,t)$, $T=\mathbb{R}$, is continuous in the mean-square sense at $t=t_0$, if $\forall \varepsilon>0$, $\exists \delta>0$, s.t.

$$\mathbb{E}|X(u,t)-X(u,t_0)|^2<\varepsilon \quad , \forall |t-t_0|<\delta .$$

Note:

1° If $R_X(t_1,t_2)$ is continuous at (t_0,t_0), since

$$\mathbb{E}|X(u,t)-X(u,t_0)|^2 = R_X(t,t)-R_X(t,t_0)-R_X(t_0,t)+R(t_0,t_0)$$

$$\leq |R_X(t,t)-R(t_0,t_0)|+|R_X(t,t_0)-R_X(t_0,t_0)| + |R_X(t_0,t)-R(t_0,t_0)|$$

Is it true from the other way around?

$\therefore$ $X(u,t)$ is continuous in the mean-square sense at t_0.

~~2° If $X(u,t)$ is w.s.s., then $R_X(t_1,t_2)=R_X(t_1-t_2)$.~~
~~When $R_X(\tau)$ is continuous at $\tau=0$, then $X(u,t)$ is continuous in the m.s.s. for all $t\in\mathbb{R}$.~~

Def: r.p. $X(u,t)$, $T=\mathbb{R}$, is uniformly continuous in the mean-square sense, if $\forall\varepsilon>0$, $\exists\delta>0$, s.t.

$$\mathbb{E}|X(u,t)-X(u,t_0)|^2<\varepsilon, \ \forall |t-t_0|<\delta, \ \forall t_0\in\mathbb{R}.$$

Note: 1° If $X(u,t)$ is w.s.s., then $R_X(t_1,t_2)=R_X(t_1-t_2)$.
When $R_X(\tau)$ is continuous at $\tau=0$, then $X(u,t)$ is uniformly continuous in the m.s.s.