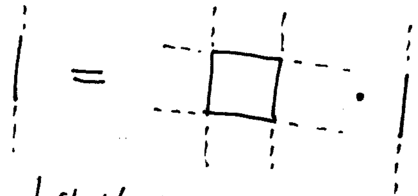
Convergence of Linear Transformation of Random Sequence. (Chap. 13·2·1)

$$X(u,t) \longrightarrow \boxed{\begin{matrix}\text{Linear}\\ \mathbb{H}\end{matrix}} \longrightarrow Y(u,t)$$

$$Y(u,t) = H \cdot X(u,t)$$

$$Y(u,t) = \sum_{t' \in \mathbb{Z}} h(t,t') \cdot X(u,t')$$



where $h(t,t')$ is time-varying impulse response of $\mathbb{H}$ at t'.

Thm: (Cauchy's Criterion)

$\forall t,\ \forall \varepsilon > 0,\ \exists N \in \mathbb{N}$, s.t. $\forall\ m,n > N$,

$$\mathbb{E}|Y_m - Y_n|^2 = \sum_{t' \in S_{m,n}} \sum_{t'' \in S_{m,n}} h(t,t') R_X(t',t'') h^*(t,t'') < \varepsilon$$

where $Y_n \equiv \sum_{t'=-n}^{n} h(t,t') X(u,t')$, $S_{m,n} \equiv \{t' : n < |t'| \le m\}$

Then $Y(u,t) = \lim_{n\to\infty} Y_n(u,t)$ exists in the m.s.s.

Thm: (A Sufficient Condition)

$\forall t,\ \forall \varepsilon > 0,\ \exists N \in \mathbb{N}$, s.t.

$$\sum_{|t'| > N} |h(t,t')|\, R_X^{\frac{1}{2}}(t',t') < \varepsilon$$

Then $Y(u,t)$ exists in the m.s.s.

Cor: If $\mathbb{H}$ is a LTI system, $h(t)$ is absolutely summable, and $R_X(t,t)$ is uniformly bounded,
then $Y(u,t)$ exists in the m.s.s.

Special Cases:

1° If $X(u,t)$ w.s.s., and $\sum_{t' \in \mathbb{Z}} |h(t,t')| < \infty$, $\forall t$,

then $Y(u,t)$ exists in m.s.s.

2° If $X(u,t)$ w.s.s., and $\mathbb{H}$ is LTI, $\sum_{t \in \mathbb{Z}} |h(t)| < \infty$

then $Y(u,t)$ exists in m.s.s. (And is also w.s.s.)

Note: 1° If a linear transformation of a Gaussian r.p. exists in m.s.s.,

then it's also a Gaussian r.p.

2° For stable LTI operators, w.s.s. input gives w.s.s. output.