

Newton's second law in different coordinates.

$$m \frac{d\vec{r}}{dt} = \vec{F}$$

A. Cartesian Coordinate.

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

$$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$

$$m\ddot{x} = F_x$$

$$m\ddot{y} = F_y$$

$$m\ddot{z} = F_z$$

B. Cylindrical Coordinate.

① polar coordinate

$$\vec{e}_r = \cos\theta\vec{i} + \sin\theta\vec{j}$$

$$\vec{e}_\theta = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

$$\dot{\vec{e}}_r = -\sin\theta\dot{\theta}\vec{i} + \cos\theta\dot{\theta}\vec{j} = \dot{\theta}\vec{e}_\theta$$

$$\dot{\vec{e}}_\theta = -\cos\theta\dot{\theta}\vec{i} - \sin\theta\dot{\theta}\vec{j} = -\dot{\theta}\vec{e}_r$$

$$\vec{r} = r\vec{e}_r$$

$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e}_\theta$$

$$m(\ddot{r} - r\dot{\theta}^2) = F_r$$

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = F_\theta$$

② Cylindrical coordinate

$$\vec{e}_R = \cos\phi\vec{i} + \sin\phi\vec{j}$$

$$\vec{e}_\phi = -\sin\phi\vec{i} + \cos\phi\vec{j}$$

$$\vec{e}_z = \vec{k}$$

$$\dot{\vec{e}}_R = \dot{\phi}\vec{e}_\phi$$

$$\dot{\vec{e}}_\phi = -\dot{\phi}\vec{e}_R$$

$$\dot{\vec{e}}_z = 0$$

$$\vec{r} = R\vec{e}_R + z\vec{e}_z$$

$$\vec{v} = \dot{R}\vec{e}_R + R\dot{\phi}\vec{e}_\phi + \dot{z}\vec{e}_z$$

$$\vec{a} = (\ddot{R} + R\dot{\phi}^2)\vec{e}_R + (2R\dot{\phi} + R\ddot{\phi})\vec{e}_\phi + \ddot{z}\vec{e}_z$$

$$m(\ddot{R} + R\dot{\phi}^2) = F_R$$

$$m(2R\dot{\phi} + R\ddot{\phi}) = F_\phi$$

$$m\ddot{z} = F_z$$

C. Spherical coordinate

$$\vec{e}_r = \sin\theta\cos\phi\vec{i} + \sin\theta\sin\phi\vec{j} + \cos\theta\vec{k}$$

$$\vec{e}_\theta = \cos\theta\cos\phi\vec{i} + \cos\theta\sin\phi\vec{j} - \sin\theta\vec{k}$$

$$\vec{e}_\phi = -\sin\phi\vec{i} + \cos\phi\vec{j}$$

$$\begin{pmatrix} \vec{e}_r \\ \vec{e}_\theta \\ \vec{e}_\phi \end{pmatrix} = \begin{pmatrix} \dot{\theta}\cos\theta\cos\phi - \dot{\phi}\sin\theta\sin\phi, \dot{\theta}\cos\theta\sin\phi + \dot{\phi}\sin\theta\cos\phi, -\dot{\theta}\sin\theta \\ -\dot{\theta}\sin\theta\cos\phi - \dot{\phi}\cos\theta\sin\phi, -\dot{\theta}\sin\theta\sin\phi + \dot{\phi}\cos\theta\cos\phi, -\dot{\theta}\cos\theta \\ -\dot{\phi}\cos\phi, -\dot{\phi}\sin\phi, 0 \end{pmatrix} \begin{pmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{pmatrix}$$

$$\dot{\vec{e}}_r = \dot{\theta}\vec{e}_\theta + \dot{\phi}\sin\theta\vec{e}_\phi$$

$$\dot{\vec{e}}_\theta = -\dot{\theta}\vec{e}_r + \dot{\phi}\cos\theta\vec{e}_\phi$$

$$\dot{\vec{e}}_\phi = -\dot{\phi}\sin\theta\vec{e}_r - \dot{\phi}\cos\theta\vec{e}_\theta$$

$$\vec{r} = r\vec{e}_r$$

$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + r\dot{\phi}\sin\theta\vec{e}_\phi$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\vec{e}_r + (2r\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\vec{e}_\theta + (2r\dot{\phi}\sin\theta + r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\vec{e}_\phi$$

$$m(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta) = F_r$$

$$m(2r\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta) = F_\theta$$

$$m(2r\dot{\phi}\sin\theta + r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta) = F_\phi$$