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曲线坐标系

1、直角坐标 (x_1, x_2, x_3) 与曲线坐标 (q_1, q_2, q_3) 的对应关系：

$$\begin{cases} x_1 = x_1(q_1, q_2, q_3) \\ x_2 = x_2(q_1, q_2, q_3) \\ x_3 = x_3(q_1, q_2, q_3) \end{cases}$$

① 基向量 $e^o = iPH^{-1} = iQ$

其中 雅可比矩阵 $P = \begin{pmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \frac{\partial x_1}{\partial q_3} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \frac{\partial x_2}{\partial q_3} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} & \frac{\partial x_3}{\partial q_3} \end{pmatrix}$，拉梅矩阵 $H \equiv \begin{pmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{pmatrix}$

拉梅系数 $h_i \equiv \sqrt{\left(\frac{\partial x_1}{\partial q_i}\right)^2 + \left(\frac{\partial x_2}{\partial q_i}\right)^2 + \left(\frac{\partial x_3}{\partial q_i}\right)^2}$

$e^o = [e_1^o, e_2^o, e_3^o]$

② 速度 $\vec{v} = \frac{\partial \vec{r}}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial \vec{r}}{\partial q_2}\frac{dq_2}{dt} + \frac{\partial \vec{r}}{\partial q_3}\frac{dq_3}{dt} = e\dot{q} = e_0(H\dot{q})$

③ 加速度 $\vec{a} = e_0(H^{-1}P^T\dot{P}\dot{q} + H\ddot{q}) = e_0(\dot{V} + Q^T\dot{Q}V)$

其中 $Q \equiv PH^{-1}$（它是个正交矩阵），

速度坐标 $V \equiv H\dot{q}$

2、常用曲线坐标系

① 柱坐标 1° 向径 $\vec{r} = \rho\vec{\rho}^o + z\vec{k}$

2° 对应关系 $x = \rho\cos\varphi$，$y = \rho\sin\varphi$，$z = z$

3° 基向量 $[\vec{\rho}^o, \vec{\varphi}^o, \vec{k}] = [\vec{i}, \vec{j}, \vec{k}]\begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4° 速度 $\vec{v} = [\vec{\rho}^o, \vec{\varphi}^o, \vec{k}][\dot{\rho}, \rho\dot{\varphi}, \dot{z}]^T$

5° 加速度 $\vec{a} = [\vec{\rho}^o, \vec{\varphi}^o, \vec{k}] \cdot \left[\begin{pmatrix} \ddot{\rho} \\ \dot{\rho}\dot{\varphi} + \rho\ddot{\varphi} \\ \ddot{z} \end{pmatrix} + \begin{pmatrix} -\rho\dot{\varphi}^2 \\ \dot{\rho}\dot{\varphi} \\ 0 \end{pmatrix}\right]$.

② 球坐标 1° 向径 $\vec{r} = r\vec{r}^o$

2° 对应关系 $x = r\sin\theta\cos\varphi$，$y = r\sin\theta\sin\varphi$，$z = r\cos\theta$

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3° 基向量 $[\vec{r}^{\circ}, \vec{\theta}^{\circ}, \vec{\varphi}^{\circ}] = [\vec{i}, \vec{j}, \vec{k}] \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$

4° 速度 $\vec{v} = [\vec{r}^{\circ}, \vec{\theta}^{\circ}, \vec{\varphi}^{\circ}]\,[\dot{r},\ r\dot{\theta},\ r\sin\theta\,\dot{\varphi}]^T$

5° 加速度 $\vec{a} = [\vec{r}^{\circ}, \vec{\theta}^{\circ}, \vec{\varphi}^{\circ}] \left[\begin{pmatrix} \ddot{r} \\ \dot{r}\dot{\theta} + r\ddot{\theta} \\ \dot{r}\sin\theta\,\dot{\varphi} + r\cos\theta\,\dot{\theta}\dot{\varphi} + r\sin\theta\,\ddot{\varphi} \end{pmatrix} + \begin{pmatrix} -r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta \\ \dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta \\ \dot{r}\dot{\varphi}\sin\theta + r\dot{\theta}\dot{\varphi}\cos\theta \end{pmatrix} \right]$