

No.

Date

Proof of Kepler's Law III

$$T^2 = \pi ab$$

$$\therefore T^2 \frac{b^2}{a^2} = \pi a \sqrt{a} \quad (bca, a)$$

$$= \pi^2 a^3 \alpha$$
$$= \pi^2 a^3 \frac{ml^2}{K} \quad (\alpha(m, l, K))$$

$$\therefore T^2 = \frac{4\pi^2 m}{K} a^3$$

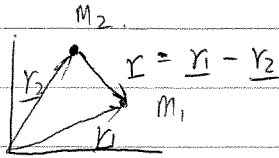
$$= \frac{4\pi^2}{GM_\odot} a^3 \quad \square$$

Yo-hoo

Yo-hoo Club, HKUSTSU, Session 09-10

two body problems.

central force.



$$\begin{cases} m_1 \ddot{\underline{r}}_1 = f(r) \hat{\underline{r}} \\ m_2 \ddot{\underline{r}}_2 = -f(r) \hat{\underline{r}} \end{cases}$$

$$\ddot{\underline{r}} = \left(\frac{1}{m_1} + \frac{1}{m_2} \right) f(r) \hat{\underline{r}}$$

$$\Leftrightarrow \frac{m_1 m_2}{m_1 + m_2} \ddot{\underline{r}} = f(r) \hat{\underline{r}}$$

$$\mu \ddot{\underline{r}} = f(r) \hat{\underline{r}}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}, \text{ reduced mass}$$

Potential energy

two particles

$$\underline{F}_{12} = \underline{F}_{12}(\underline{r}_1 - \underline{r}_2) \quad \underline{F}_{12} = -\underline{F}_{21}$$

$$\underline{F}_{12} \text{ be conservative} \Leftrightarrow \nabla_1 \times \underline{F}_{12} = 0$$

$$\underline{F}_{12} = -\nabla_1 U(\underline{r}_1)$$

$$\underline{F}_{12} = -\nabla_1 U(\underline{r}_1 - \underline{r}_2)$$

$$\therefore \nabla_1 U(\underline{r}_1 - \underline{r}_2) = -\nabla_2 U(\underline{r}_1 - \underline{r}_2) \quad (\text{chain rule})$$

$$\underline{F}_{21} = -\nabla_2 U(\underline{r}_1 - \underline{r}_2)$$

$$dE = 0$$

$$E = T + U = T_1 + T_2 + U$$

\$N\$ particles

$$U = U^{\text{int}} + U^{\text{ext}} = \sum_{\alpha < \beta} U_{\alpha\beta} + \sum_{\alpha} U_{\alpha}^{\text{ext}}$$

We are wild, exciting, challenging, fun and excellent.

$$F = T + U = \sum T_i + U$$