Ergodic Process (Chap. 16)

Concept: A w.s.s. or strictly stationary process is <u>ergodic</u>,
if the <u>time average</u> of sample function is equal to
the <u>expected value</u> of any r.v. in the process.
(or ensemble average)

Time average: $\langle X(u,t) \rangle \equiv \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} X(u,t)\, dt$

Ensemble average: $\mathbb{E}\, X(u,t)$

Def: $X(u,t)$ is <u>mean ergodic</u>, if

$$\langle X(u,t) \rangle = \mathbb{E} X(u,t), \quad \forall u \in \mathcal{U}, \forall t \in \mathbb{R}.$$

Thm: (A Sufficient Condition)

If $\mathbb{E} X(u,t) = m_X$, $\forall t$.

$$\lim_{T\to\infty} \mathbb{E} \left| \frac{1}{2T} \int_{-T}^{T} X(u,t)\, dt - m_X \right|^2 = 0$$

then $X(u,t)$ is mean ergodic.

Thm: (A Sufficient Condition 2°)

If $X(u,t)$ is w.s.s., $\lim_{\tau\to\infty} k_X(\tau) = 0$

then $X(u,t)$ is mean ergodic.

Def: $X(\omega,t)$ is <u>strictly ergodic</u>, if

$$g(X(\omega,t)) \text{ is mean ergodic}, \ \forall g(\cdot).$$

Note: 1° Strictly ergodic processes are strictly stationary

2° Any strictly stationary process is a mixture of strictly ergodic processes.