Jordan Canonical Form

Note: 1° Jordan form is not a continuous function of entries of the matrix.

E.g. $A_\varepsilon = \begin{pmatrix} \varepsilon & 1 \\ 0 & 0 \end{pmatrix}$, $\varepsilon \to 0$

2° Jordan forms are good for proofs:

(1) prove for diagonal matrices

(2) prove for Jordan blocks

(3) prove for direct [redacted] sum of matrices.

(4) prove for conjugation

Prop: (Jordan-ε form)

$A \in M_n$, $\varepsilon > 0$, then $\exists\, S =$ [redacted] $S(\varepsilon)$, s.t.

$$A = S \cdot [J_{n_1}(\lambda_1, \varepsilon) \oplus \cdots \oplus J_{n_k}(\lambda_k, \varepsilon)] \cdot S^{-1}$$

where $J_n(\lambda, \varepsilon) \equiv \begin{pmatrix} \lambda_1 & \varepsilon & & \\ & \ddots & \ddots & \\ & & \ddots & \varepsilon \\ & & & \lambda_1 \end{pmatrix}$, $\sum_i n_i = n$