Thm : (Minkowski's inequality).

A, B pos def, then

$$[\det(A+B)]^{\frac{1}{n}} \geq [\det(A)]^{\frac{1}{n}} + [\det(B)]^{\frac{1}{n}}$$

equally iff $B = cA$, some $c \geq 0$

Oct. 22 Linear programming

3 Possible cases:

(1) feasible space empty

(2) cost function unbounded in feasible space

(3) ✓ cost function has min/max in feasible space.

Thm : Optimal solutions occurs at the "corners" of the hyperspaces.

Methods to find a solution:

(0) Compute corners, evaluate cost at corners.

$\binom{m+n}{n}$ possibilities. (m: additional constrains n: variables)

(1) ~~Dantzig's~~ Dantzig's simplex method

Phase I : find a corner

Phase II : move to new corner while ~~[illegible]~~ lowering cost.

(2) Karmarkar's method

starts inside, moves to the boundary.

Min/max duality principle.

General setup:

minimize cx

subject to $x \geq 0$ & $Ax \geq b$

(n ineqs) (m ineqs)

maximize: cx

subject to: $x \geq 0$ and $Ax \leq b$

Dantzig's simplex method: $(Ax \leq b)$

<u>Def</u>: "Corner" is an intersection of n hypersurfaces that satisfies ~~satisfies all~~ all m+n inequalities.

<u>Note</u>: There are at most $\binom{m+n}{n}$ "corners."

<u>Def</u>: "edge" is an intersection of n-1 hypersurfaces bounding the feasible set.

"slack variable" $w = Ax - b$

$$\underset{m\times n}{[A} \;,\; \underset{m\times m}{I]} \underset{(m+n)\times 1}{\begin{bmatrix} x \\ w \end{bmatrix}} = b \qquad \begin{matrix} x \geq 0 \\ w \geq 0 \end{matrix}$$

new A · new x $=$ b ; new $x \geq 0$

Append 0's to cost function..

Phase I: Assume we have a corner in the feasible set, where n components of new x are 0.

<u>Def</u>: A solution is <u>basic</u> when n components of the new x are 0.

<u>Def</u>: A solution is <u>feasible</u> when new $x \geq 0$.

Def: A corner is <u>degenerate</u> if more than n components of x_{new} are 0.

Note: Degenerate corners are theoretically bad, but practically negligible.

Oct. 24

Linear programming is in P. (polynomial algorithm)

Stats w/ rescaling algorithm.

Projection: $P = I - A^T \underbrace{(AA^T)^{-1} A}_{\text{投影}}$ $\quad (A^T x = y)$

Algorithm:

1. start anywhere, construct diagonal D from x^k, s.t.
$$D^{-1} x^k = e \qquad (e = (1,1,\cdots,1))$$
2. Compute projection PDc $\qquad \left(\begin{array}{l} AD^2A^Ty = AD^2c \\ PDc = Dc - DA^Ty \end{array} \right)$
3. Find s s.t. $e - sPDc$ has zero component
4. Reduce s by a factor α (e.g. $\alpha = 0.96$)
5. New vector is $x^{k+1} = x^k - sPDc$.