Norms for vectors and matrices :

Def : (5.1.1) Vector norm $\|\cdot\|$

Nonnegative ; Positive ; Homogeneous ; Triangle inequality

Def (5.6) Matrix norm $|||\cdot|||$

Nonnegative ; Positive ; Homogeneous ; Triangle inequality ; Submultiplicative

l_p-(vector) norm : $\|\vec{x}\|_p \equiv \left(\sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}}$

Matrix norms induced by l_p vector norm : (5.6.1)

$$|||A|||_p \equiv \max_{\|\vec{x}\|=1} \|A\vec{x}\|_p$$

Special cases:

① Maximum column sum matrix norm : (5.6.4)

$$|||A|||_1 \equiv \max_{1\le j\le n} \sum_{i=1}^{n} |a_{ij}|$$

② Maximum row sum matrix norm : (5.6.5)

$$|||A|||_\infty \equiv \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|$$

③ Spectral norm : (5.6.6)

$$|||A|||_2 \equiv \sigma_1(A)$$

l_1 (Matrix) norm : $\|A\|_1 = \sum_{i,j=1}^{n} |a_{ij}|$

l_2 (Matrix) norm (Euclidean / Frobenius norm) :

$$\|A\|_2 = \left(\sum_{i,j=1}^{n} |a_{ij}|^2 \right)^{\frac{1}{2}}$$