Estimation and Testing for Hypothesis

① $\bar{X} \equiv \frac{1}{n}\sum_{i=1}^{n} X_i$ (sample mean)

$S^2 \equiv \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$ (sample variance) ; $S \equiv \sqrt{S^2}$ (sample standard deviation)

$S_p^2 \equiv \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$ (pooled estimator) ; $S_p \equiv \sqrt{S_p^2}$

② parameter : μ , σ^2/σ ; σ^2/σ $(\sigma_1=\sigma_2=\sigma)$; $\mu_1-\mu_2$

point estimator : $\bar{x}$, s^2/s ; s_p^2/s_p ; $\bar{x}_1-\bar{x}_2$

estimator : $\bar{X}$, S^2/S ; S_p^2/S_p ; $\bar{X}_1-\bar{X}_2$

③ $\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$; $\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t\text{-dist}$ $(\nu=n-1)$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2\text{-dist}$ $(\nu=n-1)$

$\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ (X_1, X_2 independent) ; $\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{S_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t\text{-dist}$ $(\nu=n_1+n_2-2)$

④ $(1-\alpha)\times100\%$ C.I. : $\mu = \begin{cases} U(\bar{x},\ z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}) & (n\geq 30) \\ U(\bar{x},\ t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}) \end{cases}$

$\sigma^2 = \left(\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}},\ \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}\right)$

$\mu_1-\mu_2 = \begin{cases} U(\bar{x}_1-\bar{x}_2,\ z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}) & \text{(sufficiently large)} \\ U(\bar{x}_1-\bar{x}_2,\ t_{\frac{\alpha}{2}}\cdot s_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}) & (\sigma_1=\sigma_2) \end{cases}$

testing for μ : $\begin{cases} a=\mu_0+z_\alpha\frac{\sigma}{\sqrt{n}} & ;\ a=\mu_0+t_\alpha\frac{s}{\sqrt{n}} \\ a,b=\mu_0\pm z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} & ;\ a,b=\mu_0\pm t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}} \end{cases}$

for σ^2 : $\begin{cases} a=\chi^2_\alpha\frac{\sigma_0^2}{n-1} & ;\ b=\chi^2_{1-\alpha}\cdot\frac{\sigma_0^2}{n-1} \\ a=\chi^2_{1-\frac{\alpha}{2}}\frac{\sigma_0^2}{n-1} & ,\ b=\chi^2_{\frac{\alpha}{2}}\frac{\sigma_0^2}{n-1} \end{cases}$

for $\mu_1-\mu_2$: $\begin{cases} a=d_0+z_\alpha\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} & ;\ a=d_0+t_\alpha\cdot s_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \\ a,b=d_0\pm z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} & ;\ a,b=d_0\pm t_{\frac{\alpha}{2}}\cdot s_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \end{cases}$