

渗流物理

助教：刘玄

Grading: Assignment 30%

Midterm 30%

Final 40%

References: 1. Petrophysics 2nd Donaldson

2. 油藏物理学基础 张世锦

3. 油层物理 何更生

4. 薛定谔

教学网上有相关资料。

L1. Reservoir Petrophysics

Petrophysics = rock property ;

rock interaction with fluids.

gases
liquid hydrocarbons (液态烃)
~~aqueous~~
aqueous solutions (水溶液)

1. properties:

Lithology 岩性.
pore
compre
perm
satur
Capillary
rock stress

2. petroleum system fluid-rock interaction

basement rock: 基岩 bottom

underburden rock: 下覆岩层

source rock: 生油层

reservoir rock: 储集层

seal rock: 密封岩

overburden rock: 上覆岩层 top

3. Sedimentary rock: 沉积岩

clastic sedimentary rock: 碎屑岩

shale 岩层

siltstone 粘土岩

sandstone 砂岩

carbon sedimentary rock: 碳酸盐岩

clastic rock: - silicate minerals (硅酸盐矿物)
- classified by: grain size (粒度), mineral composition

carbonate rock: - carbonate minerals (碳酸盐矿物)

4. sedimentary rock types: - classified by: grain size, texture

midstone 75% (泥岩): siltstone, shale

carbonate 14%

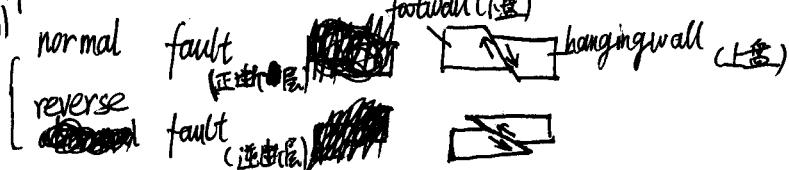
sandstone 11%

5. Grain-size classification: 石 → 砂 → 泥 → 黏土

generation
↓
migration
↓
trapping

6. anticline 背斜

structural trap (构造圈闭)



7. water drive

gas expansion drive

地层水驱动 (有一定膨胀性, 比纯水高)

气顶膨胀

溶解气分离

8. property of hydrocarbons:

色、气、密、沸、凝、flash point, #

$$\text{fluid density: } \text{API} = \frac{141.5}{\lambda} - 131.5$$

↑ ↑
API gravity specific gravity
API 重度 比重

美国标准 (STD): (atm, 15.6 °C (60°F))

drilling rigs (钻井平台): 5 种 (按海陆位置而不同)

L2: porosity

Porosity: fraction of a rock that is occupied by voids

$$\phi = \frac{V_p}{V_b} \quad - \text{pore volume}$$

- bulk volume

1. origins and descriptions

rock matrix	岩石基质
pore space	孔隙空间

original porosity	原生 ~
Induced poro~	次生 ~

classification of rocks:

sedimentary ~	: 沉积岩
igneous ~	: 火成岩
metamorphic ~	: 变质岩

compositions of rocks

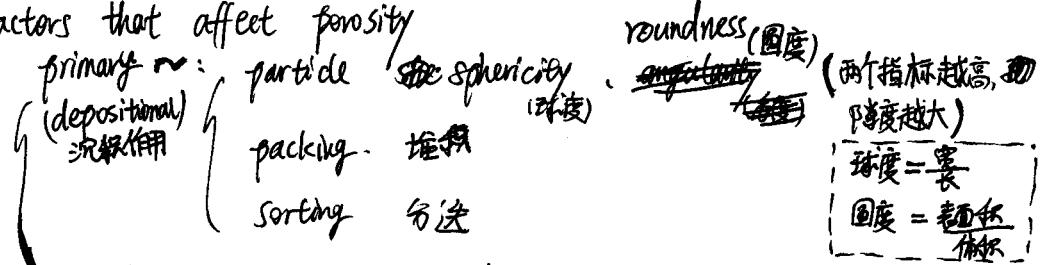
sedimentary rock	Clastic ~	sand grains
	clay matrix	chemical cement (胶结物)
	shale	sandstone
carbonate rock		

Components of sandstone:

framework	框架 (相互接触)
matrix	基质
cement	胶结物
pore	孔隙

original porosity: 原生孔隙	(primary porosity)
Induced porosity: 溶解、裂隙 (次生孔隙) (成岩作用)	(secondary porosity)

2. factors that affect porosity



secondary (diagenetic) factors:

- cementing material 胶结物质
- overburden stress 上覆压力
- vugs, dissolution, fractures (溶孔)

packing:

- tangential contact 直接接触
- sutured ~ 锯齿 ~
- long ~
- Concavo-convex ~ 凸凹接触

bedding (沉积): composition: sand, shale.

size: 层厚 slow current, fast current.

shape: 成因: eolian (风积), fluvial (河流冲积的)

orientation: 成因: river, beach

packing

destruction of bedding through bioturbation (生物扰动):

layers → homogeneous deposits

diagenesis (成岩作用) = compaction, precipitation, dissolution

dual porosity of sandstone: "matrix" porosity system, fracture porosity system

~~(dissolution porosity, 侵蚀孔隙) may be isolated~~

~~(matrix porosity, 孔隙) may be isolated~~

porosity in sandstone: $\phi <$ idealized packed sphere model.

reasons: grain size varies

grain shape varies

cementation

compaction (mechanical & chemical)

fine throat (喉道)

iron in stone will react with acid

fiber reduces poro greatly

micro-pores contain irreducible water (束缚水)

dissolution porosity (溶解孔隙): may be isolated.

Carbonate porosity types:

fabric selective: interparticle,
 intra particle,
 inter crystal,
~~moldic~~, (砂模孔隙)
 fenestral (网状孔隙)
 shelter
 growth-framework

non-fabric selective: fracture
 channel
 vug

fabric selective / non fabric selective: breccia,
 boring.
 burrow
 shrinkage

Derived Units:

SI Units are absolute and coherent (no conversion constants)

	Darcy units	SI units	Oilfield units
Time	s	s	-
Length	cm	m	ft
Mass	g	kg	lbm
Area	cm ²	m ²	ft ²
Volume	cm ³	m ³	bbl ft ³
Velocity	cm·s ⁻¹	m·s ⁻¹	-
volumetric flow rate	cm ³ ·s ⁻¹	m ³ ·s ⁻¹	bbl·day ft ³ ·day
Acceleration	cm·s ⁻²	m·s ⁻²	-
Density	g·cm ⁻³	kg·m ⁻³	lbm·ft ⁻³
Force	dyne	N	lbf
Pressure	atm	Pa	psi
Perm.	d	m ²	md
Visco. (μ)	cp	Pa·s	cp

插入R: 2~3

$$\text{注: } ① \quad 1 \text{ ft} \equiv 0.3048 \text{ m} \quad 1 \text{ ft} = 12 \text{ in}$$

$$1 \text{ lb}_m \equiv 0.453\ 392\ 37 \text{ kg} \quad (\text{中文: 磅}, \text{english: pound})$$

$$1 \text{ lbf} \equiv 1 \text{ lb}_m \cdot g_n \quad (\text{中文: 磅力}, \text{english: pound-force})$$

其中, 标准重力 standard gravity: $g_n \equiv 9.806\ 65 \text{ m}\cdot\text{s}^{-2}$

$$1 \text{ psi} \equiv \frac{1 \text{ lbf}}{1 \text{ in}^2} \approx 6894.75729 \text{ Pa} \quad \left(\begin{array}{l} \text{中文: 磅每平方英寸,} \\ \text{english: pound per square inch} \end{array} \right)$$

$$1 \text{ dyne} \equiv 1 \text{ g}\cdot\text{cm}\cdot\text{s}^{-2} \quad (\text{中文: 达因})$$

$$1 \text{ d} \equiv 1 \frac{\text{cP}\cdot\text{cm}^2}{\text{atm}\cdot\text{s}} \approx 1 \mu\text{m}^2 \quad (\text{中文: 达西}, \text{english: darcy})$$

$$1 \text{ atm} \equiv 1.01325 \times 10^5 \text{ Pa} \quad (\text{中文: 标准大气压}, \text{english: standard atmosphere})$$

$$1 \text{ P} \equiv 1 \text{ g}\cdot\text{cm}^{-1}\cdot\text{s}^{-1} = 0.1 \text{ Pa}\cdot\text{s} \quad 1 \text{ cP} = 0.01 \text{ P} = 1 \text{ mPa}\cdot\text{s}$$

$$1 \text{ bbl} \equiv 42 \text{ US gallon} \quad (\text{中文: 桶}, \text{english: oil barrel})$$

$$\approx 0.158\ 987\ 295 \text{ m}^3$$

$$\text{其中, 美制加仑 US gallon} \equiv 231 \text{ in}^3$$

② Darcy units 中, 只有压力和粘度不是 cm.g.s 制的标准值。

当然 darcy 也不是。

③ 上述 3 种单位制中, ~~SI~~ SI 单位制是 coherent (-致) 的, Oilfield 单位制不 coherent, Darcy 单位制至少对达西定律是一致的。

3. Methods of determination

- pore space classification:

$$\left\{ \begin{array}{l} \text{total porosity } \phi_t \\ \text{effective porosity } \phi_e = \frac{\text{Interconnected pore space}}{\text{bulk volume}} \end{array} \right.$$

有效孔隙中部分液体无法流动如束缚水。

- measurement of porosity

core sample (岩芯样本): 成本昂贵

openhole wireline logs (裸眼电缆测井): 测量间接、需要校正

- I. core analysis:

standard analysis:

permeability, horizontal perm,

grain density

special core analysis:

vertical perm, relative perm, capillary pressure
cementation exponent, saturation exponent

openhole 裸眼
casing 套管

裸眼完井: 井壁不稳定

水泥造成地层损伤

射孔损伤地层(压实)

- 取芯:

core bit 取芯钻头 (whole core)
sidewall sampling gun 井壁取芯枪

drill collar: 钻铤

- 岩芯样本:

whole core (全井取芯):

large, consistent, heterogeneous lithology
plug / sidewall core: (井壁岩芯)

(小岩芯) small, heterogeneous, ϕ 因施工偏大 (需校)

scalar effect (尺寸效应)

geosteering: 地质导向

- laboratory determination:

用小样本做统计分析, 需要注意采样技巧, 避免统计偏差。

$$V_b = V_m + V_p$$

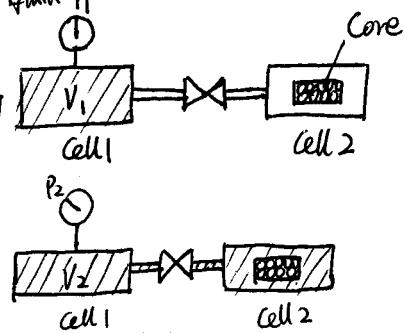
 bulk matrix pore

V_b :
 direct calculation: for regularly shaped cores/plugs ($\phi = \frac{\pi d^2 L}{4}$)
 fluid displacement, gravimetric (重力测量): archimedes method ($\phi = \frac{W_{sat} - W_{dry}}{W_{sat} - W_{sub}}$)
 volumetric (体积测量): mercury displacement Page 3

2° V_m : assume P_m matrix density : $V_m = \frac{M_m}{P_m}$
 displacement, volumetric (crush core to grain size)
 Boyles Law: $P_1 V_1 = P_2 V_2$ gravimetric $\frac{W_{dry} - W_{sub}}{\rho_{fluid} P_1} = V_m$

Comparison:

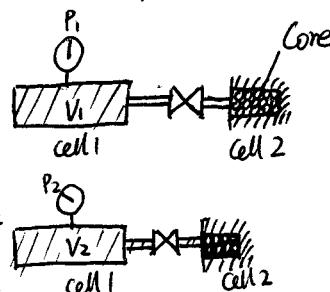
method	Accurate?	measure P ?
Assume P_m	If P_m is known	✓
Displacement	Very accurate if grains not destroyed	✗
Boyles law	Very accurate	✗



3° V_p (effective) : gravimetric (Archimedes) : $V_p = \frac{W_{sat} - W_{dry}}{\rho_{fluid}}$
 Boyle's Law : $P_1 V_1 = P_2 V_2$

Comparison:

method	Accurate?	Notes
gravimetric	if pore can be completely saturated	Fluid may react with minerals in the core
Boyle's Law	very accurate	use small M , inert gases (helium)



II. ϕ determination from logs

porosity log types: bulk density

sonic

compensated neutron (补偿中子)

STUFF BE:
0.125m

1° density log: • gamma ray back-scattered \propto electron density $\propto P_b$

P_b : bulk density (log data)

V_{sh} : volume share of shale

P_{sh} : shale density

P_{ma} : matrix density

P_{mf} : density of mud filtrate (泥浆滤液)

S_{xo} : saturation of the flush zone (冲洗带)

• To minimize the influence of mudcake (泥饼) on the wall, one should use long spacing detector (长距离探测器).

$$P_b = (1 - \phi - V_{sh}) P_{ma} + V_{sh} P_{sh} + \phi [P_{mf} S_{xo} + P_{hc} (1 - S_{xo})]$$

$$\text{If } V_{sh} \approx 0, P_{hc} \approx P_{mf} \approx P_f, \text{ then } P_b = \phi P_f + (1 - \phi) P_{ma}, \phi = \frac{P_{ma} - P_b}{P_{ma} - P_f}$$

2° neutron log: • gamma rays / captured neutrons are recorded

Δn (recorded parameter)

porosity from neutron log

P_{ma} : porosity of matrix

P_{NHC} : porosity of formation saturated

with hydrocarbon fluid

P_{Nmf} : porosity saturated with mud filtrate

• Neutron loses the most energy when collides with a hydrogen atom nucleus \Rightarrow percentage of captured neutron $\propto \phi$ (if hydrogen is in pore space), assuming calcite matrix and fresh water in pores.

$$\Delta n = (1 - \phi - V_{sh}) \phi_{Nma} + V_{sh} \phi_{Nsh} + \phi [P_{Nmf} S_{xo} + P_{Nhc} (1 - S_{xo})]$$

• elapsed time between sound wave at receiver 1 and receiver 2

$$\Delta t_L \sim P$$

$$\Delta t_L = (1 - \phi - V_{sh}) \Delta t_{ma} + V_{sh} \Delta t_{sh} + \phi [\Delta t_{mf} S_{xo} + \Delta t_{hc} (1 - S_{xo})]$$

$$\text{If } V_{sh} = 0, \text{ and hydrocarbon is liquid } (\Delta t_{hc} \approx \Delta t_{mf} \approx \Delta t_f), \text{ then } \Delta t_L = (1 - \phi) \Delta t_{ma} + \phi \Delta t_f \Rightarrow \phi = \frac{\Delta t_L - \Delta t_{ma}}{\Delta t_f - \Delta t_{ma}}$$

• factors: unconsolidated formations (松散地层) $\Delta t_f - \Delta t_{ma}$

naturally fractured formations (天然裂隙地层)

hydrocarbons (esp. gas)

rugose salt section

Δt_L : travel time read from log
(recorded parameter)

Notes:

- To determine ϕ with method II, we need logs, lithology and fluid property.
- responses of three porosity logs differ with:
 - matrix compositions
 - presence of gas or light oils
- combining 3 logs, we can know:
 - matrix compositions
 - type of hydrocarbon
- gas effect :
 - density log : ϕ too high
 - neutron log : ϕ too low
 - sonic log : ϕ not significantly affected
- advantages of log determination :
 - less expensive than coring;
 - less risky of sticking ~~the~~ tool in the hole;
 - feasible to unconsolidated formations, and formations with high secondary porosity.

L3. Compressibility of Reservoir Rocks

- compaction of sediments:
(压实)
compaction $\rightarrow \phi \downarrow$, por reduction is determined by maximum burial depth.
effects :
 - packing change (堆积)
 - pressure solution (压溶)
 - recrystallization (重结晶)
 - deformation of rock fragments (碎屑的变形)compaction is irreversible (hysteresis effect) (滞后效应).
- shale ϕ reduces more than sand stone ϕ .

• isothermal compressibility (等温压缩率)

$$C \equiv -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

where C : coefficient of isothermal compre.

unit: psi^{-1}

$C \geq 0$

V : unit: ft^3

P : unit: psi

• formation compressibility (地层压缩率)

importance: reservoir performance

{ environmental effect (subsidence) (沉降)

$$\begin{cases} \text{matrix compressibility } (C_m) : V_m, \text{ usually } C_m \approx 0 \\ \text{pore compre. } (C_p) : V_p \text{ (if for formation)} \\ \text{bulk compre. } (C_b) : V_b, \text{ usually } \partial V_b \approx \partial V_p \end{cases}$$

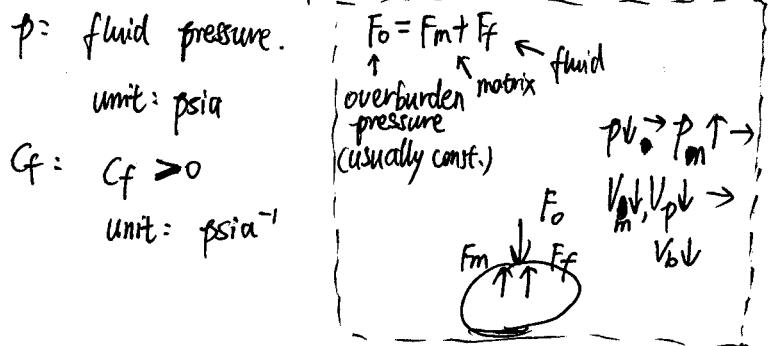
$$\text{formation compressibility} : C_f \equiv \frac{1}{V_p} \left(\frac{\partial V_p}{\partial P} \right)$$

P : fluid pressure.

unit: psi

$C_f : C_f > 0$

unit: psi^{-1}



Subsidence:

• $V_b \downarrow$, Area unchanged $\rightarrow h \downarrow$

• Net compaction pressure: P_m (matrix pressure)

$$\begin{aligned} \cancel{C_b} &= -\frac{1}{V_b} \frac{\partial V_b}{\partial P_m} = -\frac{1}{V_b} \left[\cancel{\bullet} \frac{\partial V_p}{\partial P_m} + \frac{\partial V_m}{\partial P_m} \right] \\ &= \frac{1}{V_b} (V_p C_p + V_m C_m) \\ &= \phi C_p + (1-\phi) C_m \\ &\approx \phi C_p \end{aligned}$$

calculation of V_p change:

$$\text{Integration: } C_f = \frac{1}{V_p} \frac{dV_p}{dp} \Rightarrow \int_{P_1}^{P_2} C_f dp = \int_{V_{P_1}}^{V_{P_2}} \frac{dV_p}{V_p} \Rightarrow$$

$$\Delta V_p = V_{P_1} (e^{C_f (P_2 - P_1)} - 1)$$

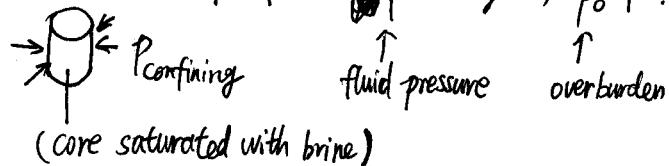
1st order approximation:

$$C_f = \frac{1}{V_p} \frac{dV_p}{dp} \approx \frac{1}{V_p} \frac{\Delta V_p}{\Delta p} \Rightarrow$$

$$\Delta V_p = V_{P_1} C_f (P_2 - P_1)$$

Laboratory determination of C_f :

- \bullet p unchanged, $p_o \uparrow$.



- \bullet $V_p \downarrow$, V_{brine} is measured.

Hysteresis Effect: (滞后效应) (压缩率值相同, 变量相反)

- \bullet effects of path dependence and irreversibilities
- \bullet Eg: $p \downarrow \rightarrow V_p \downarrow$; then $p \uparrow \rightarrow V_p \nearrow V_{p_0}$.

L4 Permeability

① permeability:

definition: measure of the capacity of the medium to transmit fluids.
 intensive property (强度量)

② permeability determination:

core analysis
 well test analysis: (repeated formation tester)
 production data:
 log data: MRI (magnetic resonance imaging)

$$\phi \propto \lg k$$

③ differential form of Darcy's law:

- $v_s = -\frac{k}{\mu} \frac{dp}{ds}$

v_s : Darcy velocity (达西速度)

$v_s = \frac{q}{A}$ ● 流量强度.

s : distance along flow path (沿流向长度)

- generalized form: $v_s = -\frac{k}{\mu} \frac{d\Phi}{ds}$

Φ : flow potential. $\Phi = p - \frac{\rho g z}{c}$, z : depth

④ Flow of liquids in porous media

1-D Linear flow

Assumption: steady state (P.T.)

incompressible fluid

$A = \text{constant}$

Darcy flow

k constant

single phase

~~μ constant~~



$$v_s = -\frac{k}{\mu} \frac{dp}{ds} \Rightarrow q \int_1^2 ds = -\frac{kA}{\mu} \int_{\Phi_1}^{\Phi_2} d\Phi$$

$$\Rightarrow q = \left(\frac{kA}{\mu L} \right) (\Phi_1 - \Phi_2)$$

1-D Radial flow

Assumption: steady state

incompressible fluid

horizontal flow

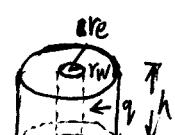
Darcy flow

k constant

single phase

~~μ constant~~

~~constant A~~



$$v_s = \frac{q}{A} = -\frac{k}{\mu} \frac{dp}{ds} \Rightarrow$$

$$\frac{q}{2\pi r h} dr = \frac{k}{\mu} \frac{dp}{ds} \Rightarrow$$

$$\frac{q}{2\pi h} \ln \frac{r_e}{r_w} = \frac{k}{\mu} (p_e - p_w)$$

$$\Rightarrow q = \frac{2\pi k h}{\mu \ln \frac{r_e}{r_w}} (p_e - p_w)$$

⑤. Gas flow in porous media

1-D linear flow:

- Assumptions: steady state (constant mass flow rate \dot{m})



$$\text{real gas law: } \rho_g = \frac{P \gamma g M_{\text{air}}}{Z RT}$$

horizontal flow ($\Phi = p$)

$A = \text{constant}$

Darcy flow

$k = \text{constant}$

Single phase

$T = \text{constant}$

$$\bullet v_s = \frac{q_g}{A} = - \frac{k}{Mg} \frac{dp}{ds}$$

$$\Rightarrow q_g ds = - \frac{KA}{Mg} dp$$

$$\Rightarrow q_{g,sc} ds = - \frac{KA}{Mg B_g} dp$$

surface condition (at $s=0$ standard condition)

$$\Rightarrow q_{g,sc} ds = - \frac{KA}{Mg} \left(\frac{T_{sc} P}{T P_{sc} Z} \right) dp \quad (Z_{sc}=1)$$

$$\Rightarrow q_{g,sc} \int_0^L ds = - KA \frac{T_{sc}}{T P_{sc}} \int_{P_1}^{P_2} \frac{p}{Z Mg} dp$$

$$\bullet \text{Integral of pressure dependent terms } I = \int_{P_1}^{P_2} \frac{p}{Z Mg} dp$$

1° "p² method": assume $Z Mg = \text{const.}$

valid if $p < 2500$ psia

for ideal gas, since $Z=1$, $Mg = Mg(T) = \text{const.}$
the assumption always holds.

$$\bullet I = \frac{1}{2ZMg} (P_2^2 - P_1^2)$$

2° Real gas pseudopressure:

$$\bullet \text{pseudopressure } m(p) = \int_{P_1}^p \frac{2p'}{Z Mg} dp'$$

$$\bullet I = \frac{1}{2} [m(P_2) - m(P_1)]$$

$$\left\{ \begin{array}{l} 1^\circ q_{g,sc} = \frac{KA}{L} \frac{T_{sc}}{T P_{sc}} \left(\frac{1}{2ZMg} \right) (P_1^2 - P_2^2) \\ 2^\circ q_{g,sc} = \frac{KA}{L} \frac{T_{sc}}{T P_{sc}} \left(\frac{1}{2} \right) [m(P_1) - m(P_2)] \end{array} \right.$$

$$2^\circ q_{g,sc} = \frac{KA}{L} \frac{T_{sc}}{T P_{sc}} \left(\frac{1}{2} \right) [m(P_1) - m(P_2)]$$

1-D Radial flow

1° "p² method"

$$q_{g, sc} = \frac{2\pi h k}{\ln \frac{r_e}{r_w}} \cdot \frac{T_{sc}}{P_{sc} T} \cdot \frac{1}{2} \cdot \frac{P_e^2 - P_w^2}{\pi \mu g}$$

2° real gas pseudopressure

$$q_{g, sc} = \frac{2\pi h k}{\ln \frac{r_e}{r_w}} \cdot \frac{T_{sc}}{P_{sc} T} \cdot \frac{1}{2} \cdot [m(p_e) - m(p_w)]$$

④ laboratory determination of ~~perm.~~ perm.

- Whole - core method

diameter: 2.5 ~ 5.5 in

length: in ~ ft

most applicable for heterogeneous formations

expensive

- Core plug method

diameter: 0.75 ~ 1.5 in

length: ~~core~~ ~ 1 in

only for homogeneous formations

most common

Note: in heterogeneous strata, core plugs should be taken

horizontal / ~~vertical~~ vertical to the strata;

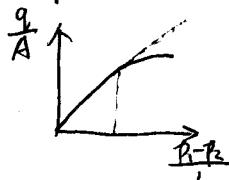
whole - core analysis is needed in case of vugs, fractures and complex lithologies.

- procedure : 1) CUT (plugs or whole-core)

2) CLEAN & Dry (extract reservoir fluids)

3) MEASURE (inlet & outlet pressure at different

- flow curve : draw $\frac{q}{A}$ versus $\frac{P_i - P_o}{L}$ flow rates



At regular speed, flow is laminar, and the curve is a straight line with slope of $\frac{k}{\mu L}$;

At very high flow rates, turbulent flow occurs, with less increase in flow rate when pressure gradient increases.

- Some issues:

- 1) core handling processes

core handling, cleaning, clay damage in drying, freezing in storage, sampling

- 2) fluid - rock interactions

clay swelling from fresh water, pore throats plugging from fines (细颗粒) migration, (堵塞)

It's preferable to use reservoir fluids or synthetic (合成的) reservoir fluids.

- 3) pressure

core alterations from loss of confining pressure (围压),

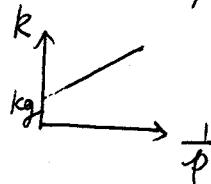
may apply a range of overburden pressures.

- 4) core heterogeneities

fracture permeability is not measured in plug method,

core mineralogy problems (salts, gypsum, 石膏)

- 5) gas velocity effects



When using gas (e.g. Nitrogen) in measuring perm., gas slippage velocity becomes important in pore space which is about the mean free path of the gas, then permeability is apparently dependent on pressure. This effect is esp. important in low-perm. rock, in low flow rates.

- Mean pressure method

$$\text{For linear flow, } q_{g,sc} = \frac{kA}{L} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{z Mg} \right) (p_i^2 - p_2^2)$$

$$\text{From real gas law, } q_{g,sc} = \frac{\bar{q} \bar{p} T_{sc}}{\bar{z} p_{sc} T}, \text{ where } \bar{p} = \frac{1}{z} (p_i + p_2) \quad (\bar{z} = z(\bar{p}, T))$$

$$\text{Then we get } \bar{q} = \frac{kA}{L} \frac{\bar{z}}{z Mg} (p_i - p_2).$$

$$\text{Let } \bar{Mg} = \frac{z Mg}{\bar{z}}, \text{ we have}$$

$$\bar{q} = \frac{kA}{\bar{Mg} L} (p_i - p_2)$$

and \bar{q} is calculated with the previous equation.

⑦ Non-Darcy flow (非达西流)

1) Gas slippage

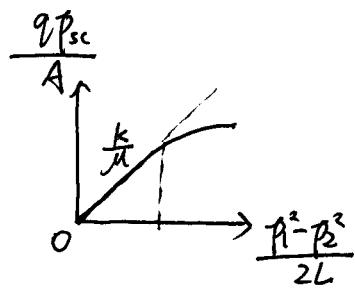
At low mean flowing pressure, gas slippage appears.

Flow at wall is not zero.

$$\text{Klinkenberg correction ratio} : k_{\text{air}} = \frac{k_{\text{ro}}}{k_A |_{P_{\text{mean}} = 1 \text{ atm.}}}$$

Klinkenberg effect is more obvious for low absolute perm. k_{ro} .

2) Turbulence



Gas wells exhibit non-Darcy flow at high flow rates.

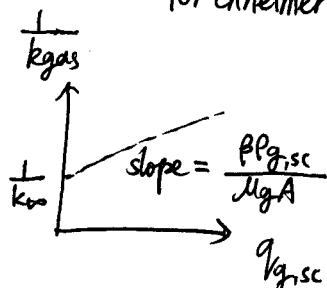
Causes: ~~inertia~~ inertial effect: velocity cannot be seen as unchanged; acceleration in pore throat, deceleration in pore body.

$$\text{Forchheimer equation: } -\frac{dp}{ds} = \frac{Mg}{k} \left(\frac{q_g}{A} \right) + \beta p_g \left(\frac{q_g}{A} \right)^2$$

~~($\beta = \beta(k_{\text{ro}})$, is empirically determined.)~~

$$\text{Forchheimer plot: Assume } -\frac{dp}{ds} = \frac{Mg}{k_{\text{gas}}} \frac{q_g}{A}$$

with Forchheimer equation, we get



$$\frac{1}{k_{\text{gas}}} = \frac{1}{k_{\text{ro}}} + \frac{\beta p_g}{Mg A} q_g$$

$$\left(= \frac{1}{k_{\text{ro}}} + \frac{\beta p_{g,sc}}{Mg A} q_{g,sc} \right)$$

where k_{ro} is the abs. perm.

⑧ Average perm. in layered systems.

- Simple cases of heterogeneity:

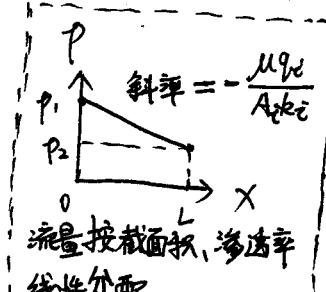
Linear, parallel — cores, horizontal perm. → 

Linear, serial — cores, vertical perm. → 

Radial, parallel — reservoir, ~~horizontal~~ horizontal layers → 

Radial, serial — reservoir, damage / stimulation (损伤/增产) → 

- Linear, parallel flow

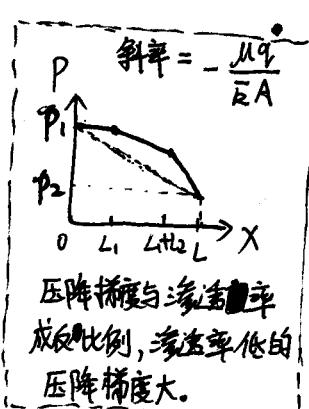
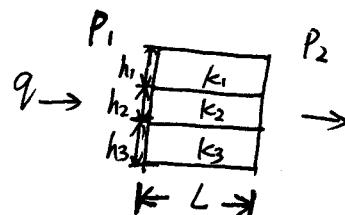


$$q_i = (h_i b) \frac{k_i}{\mu} \frac{\Delta P}{L}$$

$$q = \sum_i q_i, h = \sum_i h$$

$$q = (hb) \frac{\bar{k}}{\mu} \frac{\Delta P}{L}$$

$$\Rightarrow \bar{k} = \frac{\sum h_i k_i}{h} \quad (\text{算术平均})$$



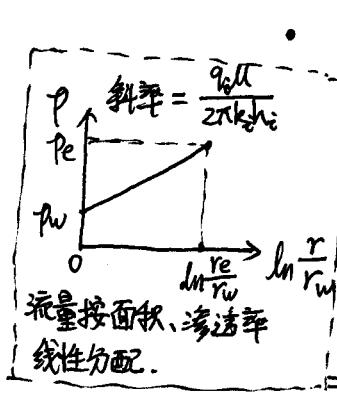
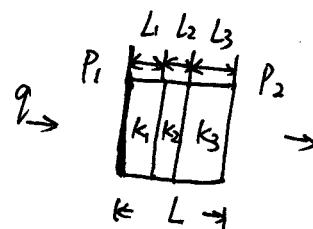
Linear, serial flow

$$q = A \frac{k_i}{\mu} \frac{\Delta P_i}{L_i}$$

$$\Delta P = \sum_i \Delta P_i, L = \sum_i L_i$$

$$q = A \frac{\bar{k}}{\mu} \frac{\Delta P}{L}$$

$$\Rightarrow \bar{k} = \frac{L}{\sum_i \frac{L_i}{k_i}} \quad \text{or} \quad (\bar{k})^{-1} = \frac{\sum L_i k_i^{-1}}{L} \quad (\text{调和平均})$$



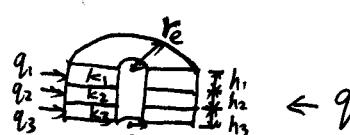
Radial, parallel flow

$$q_i = \frac{2\pi k_i h_i}{\mu \ln \frac{r_o}{r_w}} (P_e - P_w)$$

$$q = \sum_i q_i, h = \sum_i h_i$$

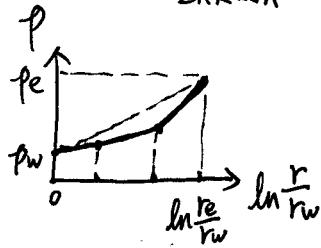
$$q = \frac{2\pi \bar{k} h}{\mu \ln \frac{r_o}{r_w}} (P_e - P_w)$$

$$\Rightarrow \bar{k} = \frac{\sum h_i k_i}{h} \quad (\text{算术平均})$$



- Radial, ~~parallel~~ serial flow

$$\frac{q}{\mu} = \frac{q}{2\pi k h}$$



压降梯度与流速率成反比例。

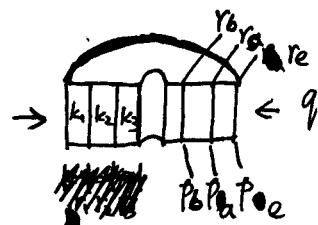
$$q = \frac{2\pi k_1 h}{\mu \ln \frac{r_w + r_1 + r_2 + r_3}{r_w + r_2 + r_3}} (P_e - P_a)$$

$$q = \frac{2\pi k_2 h}{\mu \ln \frac{r_w + r_2 + r_3}{r_w + r_3}} (P_a - P_b)$$

$$q = \frac{2\pi k_3 h}{\mu \ln \frac{r_w + r_3}{r_w}} (P_b - P_w)$$

$$q = \frac{2\pi k h}{\mu \ln \frac{r_e}{r_w}} (P_e - P_w)$$

$$\Rightarrow \bar{k} = \frac{\sum \Delta (\ln r)_i k_i}{\Delta (\ln r)} \quad (\text{averaged})$$



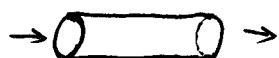
$$\text{where } \Delta (\ln r) = \ln \left(\frac{r_e}{r_w} \right), \Delta (\ln r)_1 = \ln \frac{r_e}{r_a}, \Delta (\ln r)_2 = \ln \frac{r_a}{r_b}$$

$$\Delta (\ln r)_3 = \ln \left(\frac{r_b}{r_w} \right),$$

$$\text{and } \Delta (\ln r) = \sum_i \Delta (\ln r)_i$$

⑨ flow in channels and fractures analogies to Darcy's flow

- Channels (管道流)



$$\text{Darcy's law: } q = \frac{KA}{\mu L} \Delta p$$

$$\text{Poiseuille's equation: } q = \frac{\pi r^4}{8\mu L} \Delta p = \frac{A(\frac{d^2}{32})}{\mu L} \Delta p \quad (k = \frac{d^2}{32})$$

$$\Rightarrow \text{analogously, } k \approx 2.0428 \times 10^{10} d^2 \quad (k: \text{md}, d: \text{in})$$

- Fractures (裂隙流)

$$\text{Darcy's law: } q = \frac{KA}{\mu L} \Delta p$$

$$\text{Buckingham's equation: } q = \frac{Ab^2}{12\mu L} \Delta p \quad (k = \frac{b^2}{12})$$

$$\Rightarrow \text{analogously, } k \approx 5.4476 \times 10^{10} d^2 \quad (k: \text{md}, d: \text{in})$$

- average porosity and perm.

$$\bar{\phi} = \frac{\int_V \phi dV}{V_b} = \frac{\phi_{c,f} V_{b,c,f} + \phi_{\text{matrix}} V_{b,\text{matrix}}}{V_{b,\text{total}}}$$

$$\bar{k} = \frac{\int_A k dA}{A} = \frac{k_{c,f} A_{c,f} + k_{\text{matrix}} A_{\text{matrix}}}{A}$$

(Assume parallel flow.)

L5. Saturation

- definition: fracture of pore volume.

$$S_{\text{phase}} = \frac{V_{\text{phase}}}{V_{\text{pore}}}$$

notes: 1. an intensive property

2° reservoir conditions \leftrightarrow in situ

- fundamental relationships

$$\left. \begin{array}{l} S_w + S_o + S_g = 1 \\ m_{\text{fluid}} = V_p [P_w S_w + P_o S_o + P_g S_g] \end{array} \right\}$$

- initial fluid saturations

minimum interstitial water saturation:

(油侵排水)

irreducible wetting phase saturation	(束缚水)	(drainage)
residual non-wetting phase saturation	(残余水)	(imbibition)

- determination

direct measurement:

core analysis

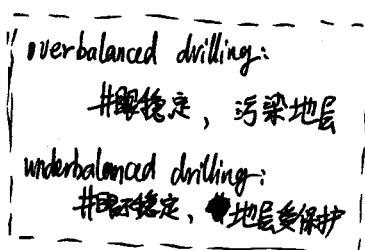
indirect measurement:

capillary pressure measurement

well log analysis (\therefore electrical conductivity $\sim S_w$)

- Core analysis (laboratory determination)

factors: 1) filtrate (滤液侵入)



Water filtrate { water based mud
oil emulsion mud (油基泥浆)

oil filtrate { oil based mud

inverted oil emulsion mud

gas filtrate { air drilling

foam drilling

2) Change in P and T during tripping from RC to SC

Eg. 1) oil zone at minimum interstitial water saturation,
water based drilling mud

	flushing	trip to surface	saturation at surface
S_w	↑	↓	probably ↑
S_o	↓	↓	↓
S_g	-	↑	↑

2) oil zone at minimum interstitial water saturation,
oil based drilling mud

	flushing	trip to surface	saturation at surface
S_w	-	-	-
S_o	-	↓	↓
S_g	-	↑	↑

estimating fluid contact depths:

gas zone $S_o \approx 0$

oil production zone $S_o \geq 0.15$

water production zone $0 \leq S_o \leq 0.15$

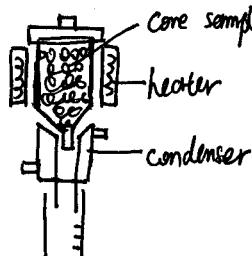
Notes:

1) The upper limit of total water for oil and ~~water~~ gas production increases, as permeability decreases.

2) Core saturations are usually not reliable for saturations within the reservoir.

3) Core saturations are very useful in other uses.

Common methods:



1) Retort distillation (蒸餾)

measured parameters: V_w , V_o

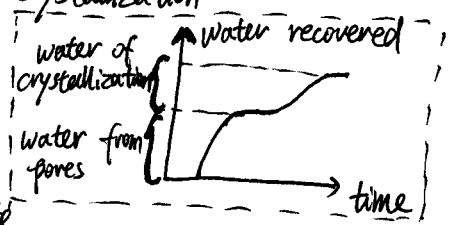
$$S_w = \frac{V_w}{V_p}, S_o = \frac{V_o}{V_p}, S_g = 1 - S_w - S_o$$

where $V_p = V_b \phi$.

Advantages: rapid (<1h); direct; adequate accuracy. | Dis: core samples destroyed by high T; crystallized water may vaporize; oil cracks into small hydrocarbons that do not recondense;

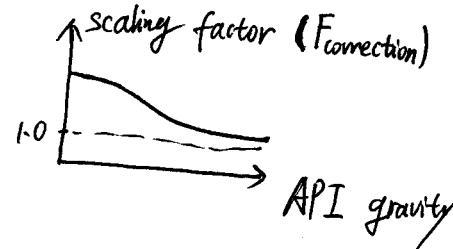
notes: 1) determining water of crystallization

if flattening of curve
is not apparent,
estimating water from
pores can be inaccurate.



2) effects of coking and cracking (焦化; 裂解)

coke formation destroys
core sample.



2) Solvent extraction (溶剂萃取)

measured parameters:

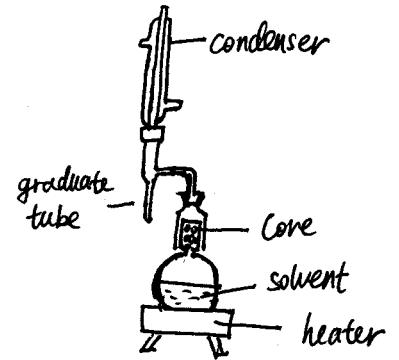
V_w , W_i , W_{dry}

initial core weight core weight after leaching

$$S_w = \frac{V_w}{V_p}$$

$$S_o = \frac{(W_i - W_{dry}) - V_w P_w}{P_0 V_p}$$

$$S_g = 1 - S_w - S_o$$



Advantages: S_w is accurate;

core sample is not destroyed;

core sample cleaned at the same time.

Disadvantages: slow (days);

oil volume measurement is indirect, since oil remains in solvent.

- electrical properties of reservoir rocks
(well log analysis)

1) resistivity: reflect the electrical flow capacity of the rock;
(电导率) (R)
an intensive property. ($\Omega \cdot m$)

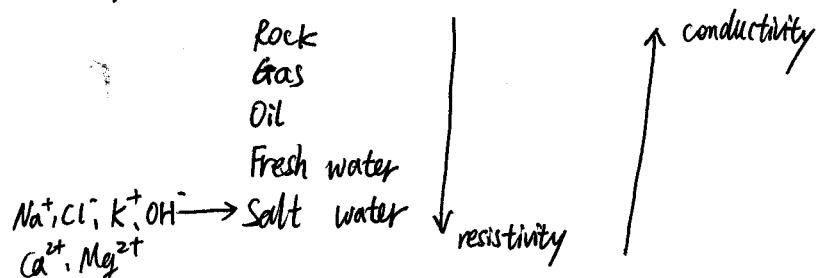
resistance: for linear, 1-D electrical flow;
(电阻) (r) extensive. (Ω)

$$r = \frac{R \cdot L}{A} \Leftrightarrow R = \frac{r \cdot A}{L}$$

conductance: inverse of resistance.
(电导) ($S = \Omega^{-1}$, Siemens)

conductivity
(电导率) = inverse of resistivity
($S \cdot m^{-1}$)

2) resistivity of earth materials:



3) factors

resistivity of water (R_w)

porosity of formation (ϕ)

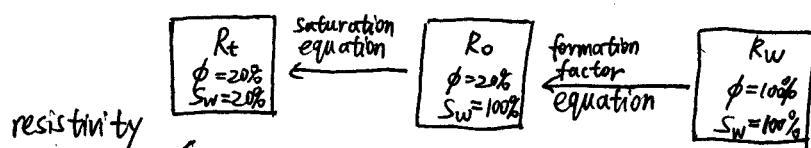
tortuosity (迂曲度)

lithology

degree of cementation (\because clay can conduct electricity.)

type and amount of clay in the rock.

4) Common notations:

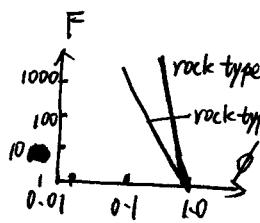


R_t : true formation resistivity

R_o : resistivity of non-shaly rock saturated with formation water

5) Archie's Law (阿奇公式)

$$\text{formation factor} : F \equiv \frac{R_o}{R_w}$$

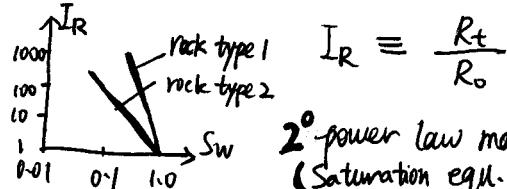


1° Archie's equation:
(formation factor eqn.)

$$F = a \cdot \phi^{-m} \quad (m > 0)$$

where $a \approx 1.0$

$m \approx 2$, cementation factor (压实因子)



$$I_R \equiv \frac{R_t}{R_o}$$

2° power law model: $I_R = S_w^{-n}$
(Saturation eqn.)

note: each curve for a specific core sample;
assume no other conductive materials (clay) present.

3° Archie's equation. (Combined)

$$S_w = \sqrt[n]{\frac{a R_w}{\phi^m R_t}}$$

where: R_w : formation water resistivity

R_t : true formation resistivity

6) Effect of Filterate Invasion (滤液侵入)

Between wellbore and uninvaded zone, are mud cake, invaded zone and transition zone, all because of filtrate invasion.

Since mud conducts electricity, the near-well resistivity is not equal to the true formation resistivity (R_t).

Logs that reach deeper into the formation can give better R_t .

L6. Capillary pressure

① Boundary tension and wettability ~~immiscible phases~~

- Interfacial tension (boundary tension) : 界面张力

definition: energy per unit area, or
force per unit distance

unit: $N \cdot m^{-1}$, or $\text{dyne} \cdot \text{cm}^{-1}$

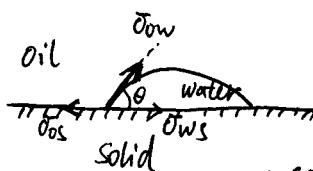
force balance: cohesive force &
adhesion force

- Wettability: (润湿性)

definition: the tendency of one fluid to spread on or adhere to a solid surface in the presence of other immiscible fluids.

- Adhesion tension (附着力): A_T

definition: the difference between two solid-fluid interfacial tensions.



$$(A_T \equiv \sigma_{os} - \sigma_{ws} = \sigma_{ow} \cos \theta) \quad (A_T \text{ between oil and water.})$$

$A_T > 0$, denser phase wet ; $A_T < 0$, lighter phase wet.

- contact angle (接触角): contact angle is measured through the denser liquid phase.

- wetting phase fluid vs. nonwetting phase fluid

in small pores

less mobile

\geq irreducible wetting phase saturation

in largest pores

more mobile

note: natural gas is never the wetting phase in hydrocarbon reservoirs.

- Water-wet reservoir rock vs. Oil-wet reservoir rock

$$\sigma_{ws} \leq \sigma_{os}$$

$$A_T \geq 0$$

$$0^\circ \leq \theta \leq 90^\circ$$

$$\sigma_{os} \leq \sigma_{ws}$$

$$A_T \leq 0$$

$$90^\circ \leq \theta \leq 180^\circ$$

- a general comparison between silicate minerals and carbonate minerals

Silicate minerals		carbonate minerals
acidic surface		basic surface
repel acidic compounds		attract acidic compound
neutral to oil-wet		neutral to oil-wet

- wettability affected by:

composition of pore-lining minerals (附着在孔隙表面的矿物)

composition of the fluids (指的是第三相)

Saturation history (影响固体表面性质)

- wettability affects:

Inducible water saturation

Residual oil and water saturation

Relative permeability k_r (各相占据不同大小的孔道, 流动性不同)

Capillary pressure P_c

Electrical properties

- classification:

strongly oil- or water-wetting

neutral wettability

fractional wettability : ~~a small portion of area~~ have different wettability

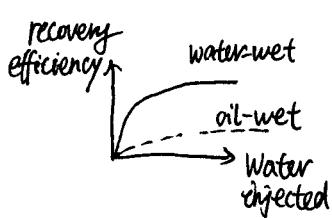
mixed wettability: Small pores are water-wet, large pores are oil-wet.

- imbibition: saturation of the wetting phase increases.

drainage: saturation of the wetting phase decreases.

Note: 1° 某一相的饱和度越大, 占据的孔道就越多, 相渗透率就要增加。(尽管不是线性的.)

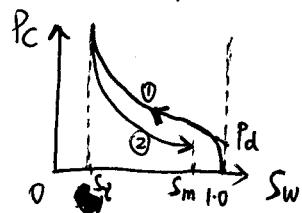
- Implications of wettability



- 1) A water-wet system exhibits greater primary oil recovery.
- 2) A water-wet system exhibits greater oil recovery under waterflooding.
- 3) Wettability affects the shape of the relative permeability curves. (水润湿法)

② Introduction to Capillary pressure

- Capillary pressure curves



①: Drainage P_c curve

②: Imbibition P_c curve

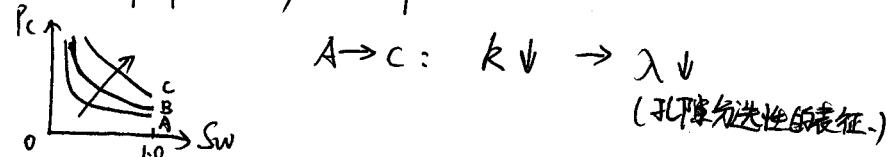
~~Sw~~: irreducible water phase saturation

$S_m = 1 - \text{residual non-wetting phase saturation}$

P_d : displacement pressure

(非润湿相进入最大毛孔的毛管力).

- Effects of permeability on shape



$$A \rightarrow C : k \downarrow \rightarrow \lambda \downarrow$$

(孔隙率与透性的特征.)

- Effects of grain size distribution on shape

$$\text{Sort} \downarrow \rightarrow \lambda \downarrow$$

- Capillary pressure: The pressure difference existing across the interface separating two immiscible fluids in capillaries.

$$P_c = P_{nw} - P_w$$

- Height of water rise in Capillary tube: (Air/Water system)

$$\Delta h = \frac{2 \sigma_{aw} \cos \theta}{r \Delta P_{aw} g}$$

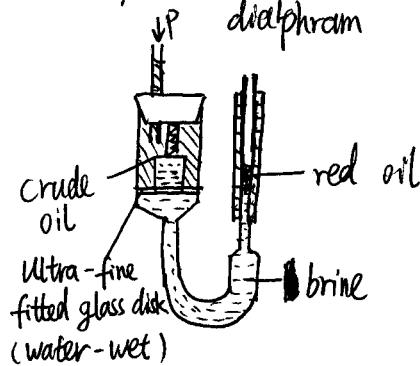
where: σ_{aw} — interfacial tension between air/water.

- Capillary pressure of air/water system.

$$P_c = \frac{2\sigma_{aw} \cos \theta}{r}$$

③ Laboratory measurement of capillary pressure

- porous ~~disk~~ method (多孔隔膜法)



Advantages:

1. very accurate
2. use reservoir fluids

Disadvantages:

1. very slow
2. measured capillary pressure \leq "displacement pressure" of the disk.
("displacement pressure": minimum capillary pressure of the disk.)

- Mercury injection method

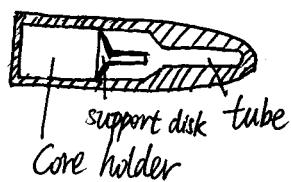
Advantages

1. quick
2. reasonably accurate
3. very high range of capillary pressures

Disadvantages

1. ruins core
2. hazardous testing material
3. need converse the mercury/air capillary data to reservoir fluid systems.

- centrifuge method



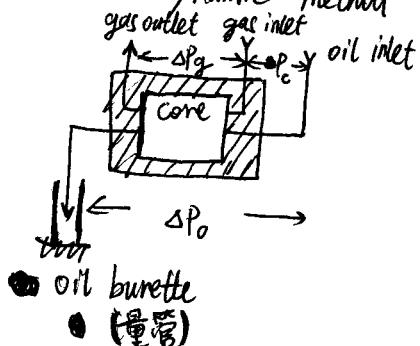
Advantages

1. fairly quick
2. reasonably accurate
3. use reservoir fluids

Disadvantages

1. complex analysis

- Dynamic method



Advantages

1. simulates reservoir flow
2. use reservoir fluids

Disadvantages

1. very tedious
2. high cost.

~~Saturation history and Capillary pressure in reservoir rock.~~

④ J-function

- The rock properties affecting capillary pressures in reservoir have extreme variation with lithology.

But J-function is valuable for correlating capillary pressure data within a lithology.

- Definition: $J(S_w) = \frac{C P_c}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}}$

where, C — unit conversion factor
J-function is dimensionless;

for a particular rock type, J-function is the same for any unit system, any two fluids and any exact value of k, ϕ .

- use of Leverett J-function:

averaging capillary pressure data from a given rock type from a given reservoir;

can sometimes be extended to different reservoirs having same lithologies;

not accurate for different lithologies;

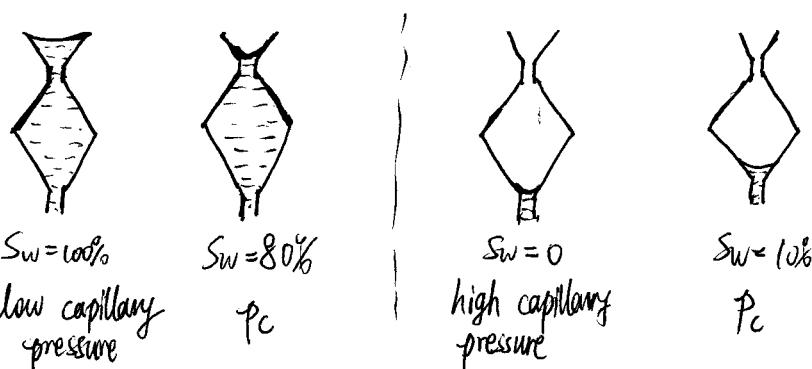
failure of J-function implicates variation in rock type.

在 J 函数的适用范围内, J 函数与 P_c 曲线只差一个常数项。

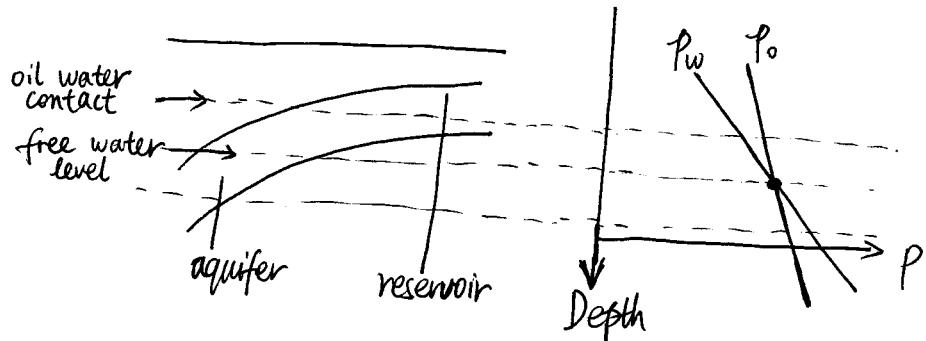
⑤ Saturation history and capillary pressure in reservoir rock.

- Saturation history

Capillary pressure depends on both direction of change (Imbibition/drainage) and previous saturation history.



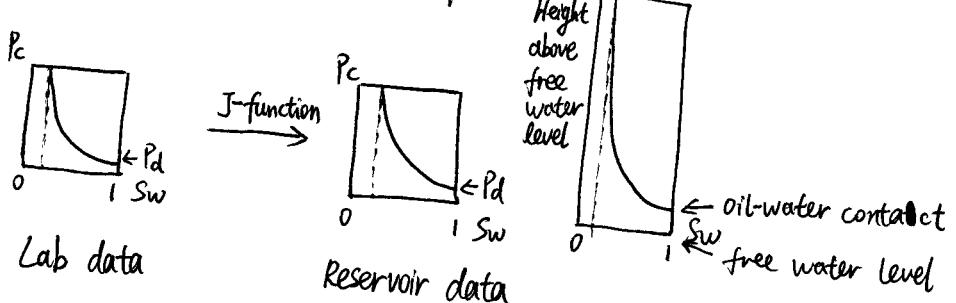
- Capillary pressure in reservoirs



free water level : $P_c = 0$

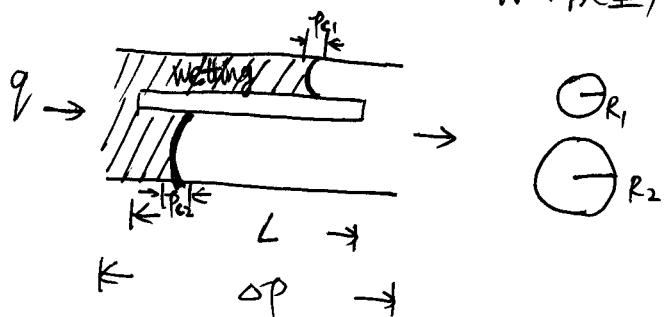
oil water contact : $P_c = \min \{ P_c \text{ in reservoir rock} \}$

- Saturation in reservoirs vs. depth



⑥ S_{or}, S_{wi} models

- pore doublet model (孔隙对模型)



Assumptions:

- 1° Poiseuille flow describes the flow of each fluid in each ~~wetting~~ capillary.

- 2° $\mu_{\text{wetting}} = \mu_{\text{non-wetting}}$

Derivation: According to Poiseuille flow

$$q_1 = \frac{\pi R_1^4}{8\mu L} (\Delta P + p_{c1})$$

$$q_2 = \frac{\pi R_2^4}{8\mu L} (\Delta P + p_{c2})$$

and capillary pressure in tubes

$$p_{c1} = \frac{2\sigma \cos\theta}{R_1}$$

$$p_{c2} = \frac{2\sigma \cos\theta}{R_2}$$

We express each equation in terms of q , where $q = q_1 + q_2$, instead of ΔP .

Then the relationship between velocities ($V = \frac{q}{\pi r^2}$) in each capillary is:

$$\frac{V_2}{V_1} = \frac{4N_{cap} + \frac{1}{\beta} - 1}{\frac{4N_{cap}}{\beta^2} - \beta^2 \left(\frac{1}{\beta} - 1 \right)}$$

$$N_{cap} \sim \frac{\text{粘性力}}{\text{毛管力}}$$

这是毛管数的物理意义

where heterogeneity factor $\beta = \frac{R_2}{R_1}$

$$\text{local capillary number } N_{cap} = \frac{\mu L q}{\pi R_1^3 \sigma \cos\theta}$$

conclusions:

When q is large, $\frac{V_2}{V_1} \approx \left(\frac{R_2}{R_1}\right)^2$, nonwetting fluid is trapped in the small capillary.

When q is small, $\frac{V_2}{V_1} < 1$, nonwetting fluid is trapped in the large tube. This is realistic.

There is no trapping without heterogeneity.

- Snap-off model



L7 Relative permeability

① Effective permeability & relative permeability

- Effective permeabilities: k_o, k_g, k_w

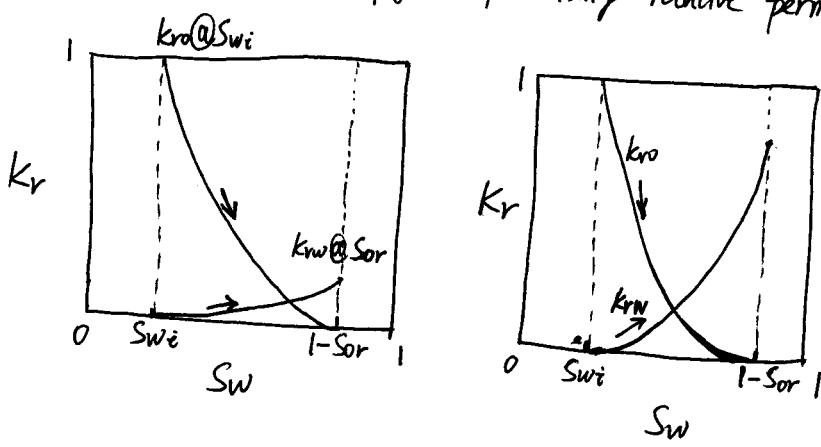
$$q_l = A \frac{k_l}{\mu_l} \frac{\Delta \Phi_l}{L} \quad (l = o, g, w)$$

Relative permeability: $k_{rl} \equiv \frac{k_l}{k_{base}}$

where the base permeability k_{base} is typically:

k_{abs} ; k_{air} ; $k_o(S_w = S_{wi})$

The base permeability should always be noted along with tables and figures presenting relative permeability data.



Strongly water-wet rock
(imbibition)

Strongly oil-wet rock
(drainage)

Note: 1° 各相之间流动相互干扰，总有效渗透率要低于绝对渗透率

2° 油藏在油水体系下， $k_o @ S_w = S_{wi} > k_w @ S_o = S_{or}$

3° k_r 与饱和度变化方向有关，不是饱和度的单值函数。

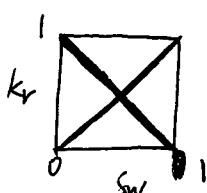
4° 油湿油藏中油的润湿性不如水湿油藏中水的润湿性强。

5° 一般情况下， k_r 以 $k_o @ S_w = S_{wi}$ 为分母。

6° 相渗曲线为凸曲线，因为 1) 相饱和度大时为连续相，易流动；

2) 相先后占据大小孔道，流动性不一。

7° 在裂缝、大孔道里，流速大，毛管压力影响小，润湿性效果不明显，相对渗透率曲线形式如左。

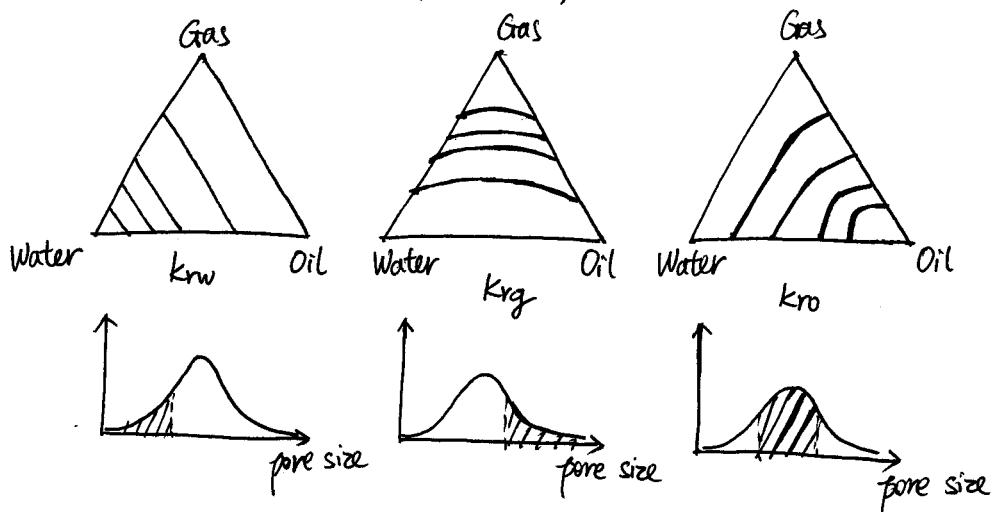


Factors affecting relative permeability:

{ Saturations,
 wettability,
 Fluid saturation history,
 Geometry of pore spaces and pore size distribution

Relative permeability is unique for different rocks and fluids.

- Three phase relative permeability



Function only of
water saturation.

Function primarily of
gas saturation.

Function of both water
and gas saturation.

When S_g is small, k_{ro} is
primarily a function of S_o ;
when S_w is small, k_{ro} is
primarily a function of S_g .

Three phase flow ($k_{ro}, k_{rw}, k_{rg} \geq 0.05$) occurs over a limited range of three phase saturations.

Stone's Method II (Calculation of three phase relative permeability):

$$k_{rg} = k_{rg}(S_g) \text{ on gas-oil } \text{---} \text{ curve;}$$

$$k_{rw} = k_{rw}(S_w) \text{ on oil-water curve;}$$

$$k_{ro} = (k_{rw} + k_{rg})(k_{rog} + k_{rgo}) - k_{rgo} - k_{wo};$$

where k_{rw} means k_{ro} from o-w curve, ---.

这是以前的
一种计算模型，
精度也未很好。

② Laboratory Measurement of relative permeability

- Steady state method

- 1) Simultaneously inject constant rates of oil and water, until steady state behavior (q_o, q_w) is observed. (if $q_o \neq q_w$)
- 2) determine saturation of core sample, usually by resistivity or weighing.
- 3) change injection ratio and repeat.

Some preliminary steps:

saturate the core with wetting phase;

determine absolute permeability;

inject non-wetting phase until steady state, measure saturation and effective permeability.

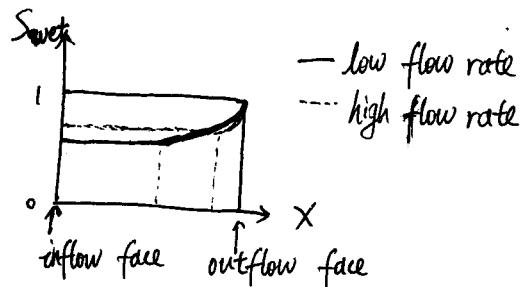
- Capillary end effect

During immiscible displacement, at the ~~outflow~~ outflow face

$P_c = 0$, according to $P_c(S_{wet})$, we get $S_{wet} = 1$.

However, in the bulk of the core plug, saturation can vary.

There must be a gradient of saturation from the bulk to the outflow face. This saturation gradient is the "Capillary End Effect".



To eliminate errors due to ~~end~~ end effect in measurement of k_r :

- 1) measure saturation far enough away from outflow face
- 2) use high flow rates.

- Unsteady state method

Process : Saturate core with wetting phase ;

Inject non-wetting phase until steady state ,
measure saturation ;

Inject wetting phase, measure fractional flow and
cumulative non-wetting phase production , until no more
non wetting phase is produced.

Assumptions :

- 1) immiscible displacement

2) incompressible fluids

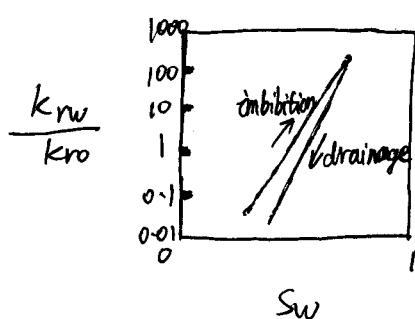
3) linear, 1-D flow

4) neglect capillary pressure . residual

Analysis :

- 1) Determine average saturation (including nonwetting phase saturation) from cumulative non-wetting phase production
- 2) Determine relative permeability ratio from fractional flow.

$$f_{\text{wet}} = \frac{1}{1 + \left(\frac{k_{r,\text{nonwet}}}{k_{rw}} \cdot \frac{M_{\text{wet}}}{M_{\text{nonwet}}} \right)}$$



L8 Buckley - Leverett Theory (含油排替法)

① Immiscible displacement equations of motion (不混溶驱替运动方程)

- Conservation of mass and momentum law

Assume: 1° porosity is constant

2° water / oil density is constant

From conservation of mass,

$$\frac{\partial(\phi p_w s_w)}{\partial t} + \nabla \cdot (p_w \vec{u}_w) = 0$$

we have (Saturation equation)

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \vec{u}_w = 0$$

$$\text{Similarly, } \phi \frac{\partial s_o}{\partial t} + \nabla \cdot \vec{u}_o = 0$$

Since $s_w + s_o = 1$, we have

$$\nabla \cdot (\vec{u}_o + \vec{u}_w) = 0$$

From Darcy's Law, we have

$$\begin{cases} \vec{u}_w = - \frac{k k_{rw}}{M_w} \nabla (P_o - P_c + p_w g h) \\ \vec{u}_o = - \frac{k k_{ro}}{M_o} \nabla (P_o + P_c g h) \end{cases}$$

Define $\lambda_w = \frac{k k_{rw}}{M_w}$, $\lambda_o = \frac{k k_{ro}}{M_o}$, $\lambda_T = \lambda_w + \lambda_o$

Then we have (pressure equation)

$$\nabla \cdot [\lambda_T \nabla P_o - \lambda_w \nabla P_c + (p_w \lambda_w + p_o \lambda_o) g \nabla h] = 0$$

Assume: 3° Capillary and gravity forces are negligible.
we get,

$$\nabla \cdot (\lambda_T \nabla P_o) = 0$$

This is a nonuniform Laplace equation for pressure.

- Fractional flow (分流量)

- Definition: $f_w \equiv \frac{q_w}{q_w + q_o}$

- Since $(\vec{u}_o + \vec{u}_w) = -\lambda_T \nabla p_o + \lambda_w \nabla p_c - (\lambda_o p_o + \lambda_w p_w) g \nabla h$

we have $\nabla p_o = -\frac{1}{\lambda_T} [(\vec{u}_o + \vec{u}_w) - \lambda_w \nabla p_c + (\lambda_o p_o + \lambda_w p_w) g \nabla h]$

With Darcy's Law,

$$\vec{u}_w = \frac{\lambda_w}{\lambda_T} [(\vec{u}_o + \vec{u}_w) - \lambda_w \nabla p_c + (\lambda_o p_o + \lambda_w p_w) g \nabla h] + \lambda_w \nabla p_c$$

$$- \lambda_w p_w g \nabla h$$

$$= \frac{\lambda_w}{\lambda_T} [(\vec{u}_o + \vec{u}_w) + \lambda_o \nabla p_c - \lambda_o (p_w - p_o) g \nabla h]$$

Hence, $f_w = \frac{\lambda_w}{\lambda_T} \left[1 + \frac{\lambda_o}{(\vec{u}_o + \vec{u}_w)} (\nabla p_c - \Delta p g \nabla h) \right]$

where $\Delta p = p_w - p_o$

Note: 1° We now only use this formula in 1D flow,
so there is no improper use of vector operation.

2° In the absence of capillary and gravity forces,

$$f_w = \frac{\lambda_w}{\lambda_T}$$

- From saturation equation $\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \vec{u}_w = 0$

we have $\phi \frac{\partial s_w}{\partial t} + \nabla \cdot [f_w (\vec{u}_w + \vec{u}_o)] = 0$

Since $\nabla \cdot (\vec{u}_w + \vec{u}_o) = 0$,

we have $\phi \frac{\partial s_w}{\partial t} + (\vec{u}_w + \vec{u}_o) \cdot \nabla f_w = 0$

Define interstitial velocity $\vec{v} = \frac{\vec{u}_w + \vec{u}_o}{\phi}$,

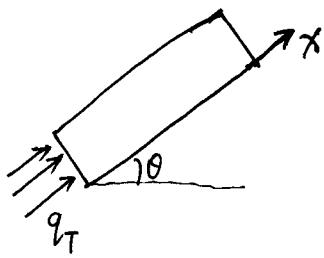
then

$$\frac{\partial s_w}{\partial t} + \vec{v} \cdot \nabla f_w = 0$$

② Buckley-Leverett Theory - 1D flow

- Equations of flow

Consider linear one-dimensional flow with constant dip angle.



The pressure equation is

$$\frac{\partial}{\partial x} \left(\lambda_T \frac{\partial \pi}{\partial x} \right) = 0 \quad (P_c \text{ ignored})$$

The saturation equation is

$$\frac{\partial S_w}{\partial t} + v \frac{\partial f_w}{\partial x} = 0$$

$$\text{where } f_w = \frac{\lambda_w}{\lambda_T} \left[1 + \frac{\lambda_w A}{q_T} \left(\frac{\partial P_c}{\partial x} - \Delta \rho g \sin \theta \right) \right]$$

Note: $f_w^0 = f_w(S_w, \theta)$

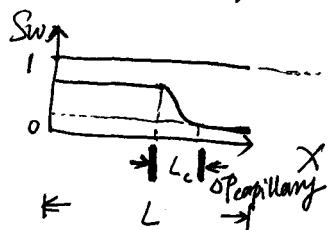
2° For flow up-dip ($\theta > 0$), gravity retards the water flow;

for flow down-dip ($\theta < 0$), gravity increases the water flow.

3° If $\frac{q_T}{A}$ is large, then the flow is viscous-dominated,
and ~~capillary~~ capillary pressure and gravity have less influence.

- Capillary pressure gradient terms. (How can we ignore it.)

Near a water/oil displacement front, S_w distribution is:



$$\text{Since } \frac{\partial P_c}{\partial x} = \frac{dP_c}{dS_w} \frac{\partial S_w}{\partial x},$$

the capillary pressure gradient is only important in the vicinity of the front.

Define local capillary-to-viscous number $N_{cv} \equiv \frac{kA \Delta P_{capillary}}{qML}$.
Since ~~in~~ the vicinity of a stable front, $\Delta P_{viscous} \approx \Delta P_{capillary}$
and $\Delta P_{viscous} = \frac{qML}{kA}$.

$$\text{So } N_{cv} = \frac{\Delta P_{capillary} \cdot L_c}{\Delta P_{viscous} \cdot L} \approx \frac{L_c}{L}$$

$\therefore N_{cv}$ can be seen as the relative size of the region where capillary ~~gradient~~ is important.

At field scale, L is very big, Ncr is small, so that the capillary region is negligible. This justifies most applications of Buckley-Leverett calculations at field scale.

However, we cannot ignore capillary pressure gradients at the lab scale.

- Fractional flow with gravity only

Ignoring the gradient of capillary pressure, we get

$$f_w(S_w, \theta) = \frac{\lambda_w}{\lambda_T} \left[1 - \frac{\lambda_o A}{q_T} \Delta P g \sin \theta \right]$$

Define gravity number $Ng = \frac{\text{gravity}}{\text{viscous forces}} = \frac{KA \Delta Pg}{\mu_0 q_T}$

$$\text{Since } \frac{\lambda_w}{\lambda_T} = \frac{1}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}},$$

We have $f_w(S_w, \theta) = \frac{1 - Ng k_{ro} \sin \theta}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}}$

Suppose $\begin{cases} k_{ro} = k_{ro}^o (1 - S_w^*)^{n_o} \\ k_{rw} = k_{rw}^o (S_w^*)^{n_w} \end{cases}$ (Honarpour's relation)

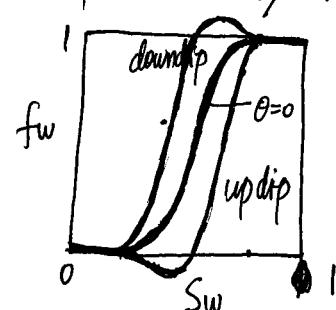
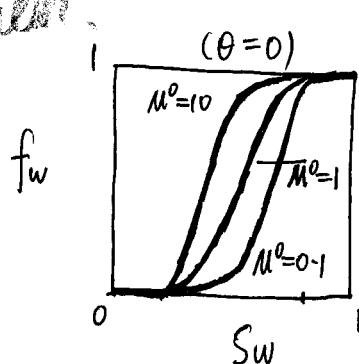
$$\text{where } S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}$$

We have

$$f_w(S_w, \theta) = \frac{1 - Ng k_{ro}^o (1 - S_w^*)^{n_o} \sin \theta}{1 + \frac{(1 - S_w^*)^{n_o}}{M^o (S_w^*)^{n_w}}}$$

where ~~$M^o = \frac{k_{rw}^o \mu_o}{\mu_w k_{ro}^o}$~~ , is the water/oil end-point mobility ratio.

Draw these:



when $f_w > 1$ or $f_w < 0$, there is countercurrent flow.

- The Buckley-Leverett Solution

For constant dip angle θ , $f_w = f_w(S_w)$, the saturation equation is

$$\frac{\partial S_w}{\partial t} + V \frac{df_w}{dS_w} \frac{\partial S_w}{\partial x} = 0$$

Define dimensionless variables

$$X_D = \frac{x}{L}$$

$$t_D = \frac{\int_0^t q_T dt}{V_{pore}}$$

Since $V = \frac{q_T}{\phi A}$, we have

$$\frac{\partial S_w}{\partial t_D} + \frac{\partial f_w}{\partial S_w} \frac{\partial S_w}{\partial X_D} = 0$$

- Method of characteristics

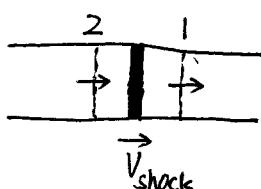
Along a curve Γ : $\frac{dx_D}{dt_D} = \frac{df_w}{dS_w}$

$$x_D(0) = x_0$$

it holds $\frac{dS_w}{dt_D}|_\Gamma = 0$.

Thus $\frac{dx_D}{dt_D}|_\Gamma = f'_w(S_w)|_\Gamma = \text{const}$, Γ is a straight line.

- Shock velocities



Consider a control volume moving with the shock at speed V_{shock} . The mass conservation equation is:

flow rate in = flow rate out

$$q_T f_w 2 - V_{\text{shock}} \phi A S_w 2 = q_T f_w 1 - V_{\text{shock}} \phi A S_w 1$$

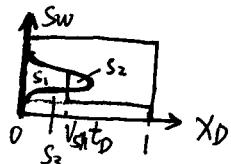
We get

$$V_{\text{shock}} = \frac{q_T}{\phi A} \frac{f_w 2 - f_w 1}{S_w 2 - S_w 1} = V \frac{\Delta f_w}{\Delta S_w}$$

Define dimensionless shock velocity as

$$V_{shD} \equiv \frac{V_{sh}}{V} = \frac{\Delta f_w}{\Delta S_w}$$

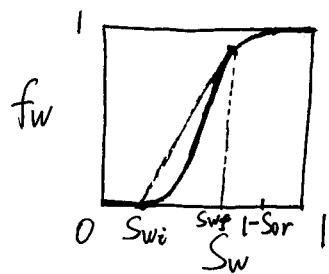
From mass conservation of water in the whole core, we can find



$$S_2 = S_3$$

This helps determine V_{shD}

- Wedge construction



$$\begin{cases} S_{WI} = S_{wi} & (f_w = 0) \\ S_{WB} = 1 - S_{or} & (f_w = 1) \end{cases}$$

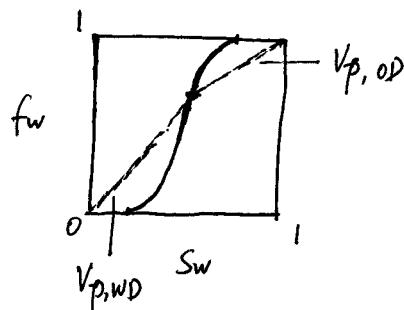
In this case, shock velocity is

$$V_{SHD} = \frac{\Delta f_w}{\Delta S_w} = \frac{f_{wf} - f_{wi}}{S_{wf} - S_{wi}}$$

- Classification of waves

$$\text{Particle velocity } V_{p,0} = \frac{\rho_T f_0}{\rho A S_0}$$

$$\text{Dimensionless particle velocity } \begin{cases} V_{p,0D} = \frac{V_{p,0}}{V_T} = \frac{f_0}{S_0} = \frac{1-f_w}{1-S_w} \\ V_{p,WD} = \frac{f_w}{S_w} \end{cases}$$



Classification:

- 1) rarefaction
- 2) shock
- 3) mixed
- 4) indifferent (neither spreads nor sharpens)

- Average saturation

After breakthrough, the average saturation at given time t_D is

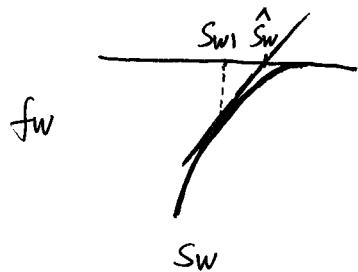
$$\begin{aligned} \hat{S}_w(t_D) &= \cancel{\int_{0}^{t_D} \int_{0}^{x_D} S_w dX_D} S'_w dX_D \\ &= \cancel{\int_{0}^{t_D} x_D S_w \Big|_0^1} - \int_{1-S_{or}}^{S_{wi}} x_D dS_w \end{aligned}$$

Since any x_D is in the spreading part of the wave, we have

$$f'_w = \frac{x_D}{t_D}$$

$$\begin{aligned} \text{Then } \hat{S}_w(t_D) &= S_{wi} - \cancel{\int_{1-S_{or}}^{S_{wi}} t_D f'_w dS_w} \\ &= S_{wi} - t_D (f'_{wi} - 1) \\ &= S_{wi} - \frac{f'_{wi} - 1}{f'_{wi}} \end{aligned}$$

$$\therefore f'_{wi} = \frac{1 - f_{wi}}{\bar{S}_w(t_0) - S_{wi}}$$



So the average saturation can be found as the figure shows.

- Oil recovery calculation

Measure :

- Cumulative oil recovery $N_p(t_0)$

Fractional flow of water in effluent $f_{we}(t_0)$

Pore volume $V_p = \phi AL$

Original oil in place (OOIP) S_{oi} .

Calculate :

$$S_{wi} = 1 - \frac{S_{oi}}{V_p}$$

$$N_{pd} = \frac{N_p}{V_p}$$

$$\bar{S}_w = S_{wi} + N_{pd}$$

After breakthrough, we have $S_{wi} = S_{we}$, $f_{wi} = f_{we}$
From last section, we get

$$\bar{S}_w = S_{we} - t_0(f_{we} - 1)$$

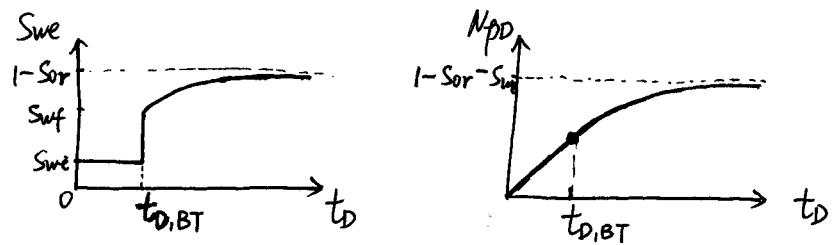
Then $S_{we} = \bar{S}_w - t_0(1 - f_{we})$

Results :

$S_{we} =$	$\begin{cases} S_{wi} & \text{(before breakthrough)} \\ \bar{S}_w - t_0(1 - f_{we}) & \text{(after breakthrough)} \end{cases}$
$N_{pd} =$	$\begin{cases} t_0 & \text{(before breakthrough)} \\ \frac{N_p}{V_p} & \text{(after breakthrough)} \end{cases}$

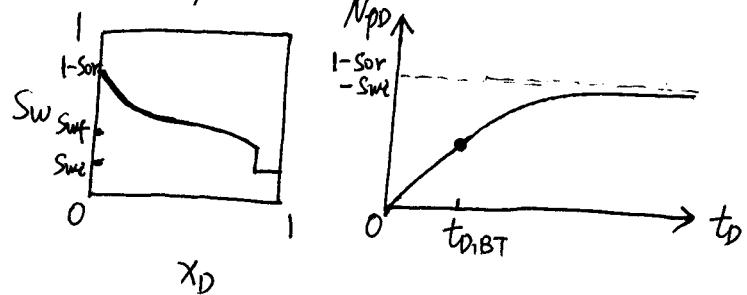
Additional relation:

$$f'_{wf} = \frac{1}{t_0} = \frac{1}{\bar{S}_w - S_{wi}}$$

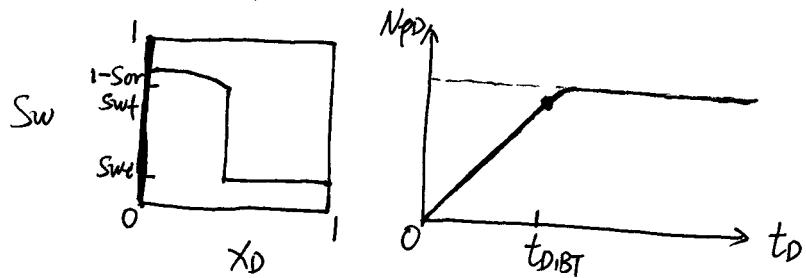


- Effect of mobility on recovery

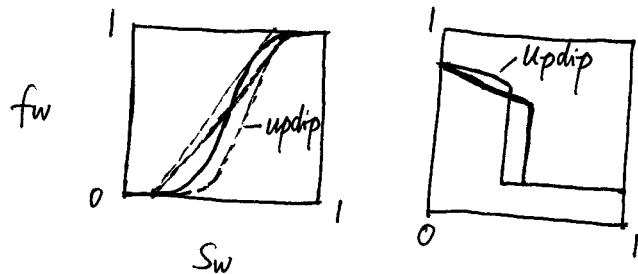
High mobility ratio (unfavorable)



Low mobility ratio (favorable)



- Effect of gravity on recovery



Updip : Increase breakthrough water saturation ;
Postpone breakthrough ;

Downdip : (The inverse)

③ Derivation of Relative Permeability from Displacements

- JBN Method

Assumptions:

- 1° Ignore capillary end effect

- 2° Ignore capillary pressure gradient region

- 3° Ignore gravity force.

Derivation: By assumption 1°, 2°, the flow is governed by Buckley - Leverett theory.

$$\therefore q_0 = q_T f_0$$

$$q_0 = - \frac{k k_{ro}}{\mu_0} A \frac{\partial p}{\partial x}$$

$$\therefore \Delta p = \frac{q_T \mu_0}{kA} \int_0^L \frac{f_0}{k_{ro}} dx$$

$$= \frac{q_T \mu_0 L}{kA} \int_0^1 \frac{f_0}{k_{ro}} dx_D$$

After breakthrough,

$$x_D = \frac{f'_w(x_D)}{f'_{we}}$$

$$\therefore dx_D = \frac{df'_w}{f'_{we}}$$

$$\therefore \Delta p = \frac{q_T \mu_0 L}{kA f'_{we}} \int_{f'_{wb}}^{f'_{we}} \frac{f'_w}{k_{ro}} df'_w$$

If we take the base permeability as k_0 @ $S_w = S_{wi}$, then at the start of the displacement,

$$q_T = q_0 = \frac{k}{\mu_0} A \frac{\Delta p}{L}$$

Define standard injectivity $(\frac{q_T}{\Delta p})_S = \frac{kA}{\mu_0 L}$

relative injectivity

$$I_R = \frac{q_T / \Delta p}{(\frac{q_T}{\Delta p})_T} = \frac{q_T \mu_0 L}{\Delta p k A}$$

Then $\int_{f'_{wb}}^{f'_{we}} \frac{f'_w}{k_{ro}} df'_w = \frac{f'_{we}}{I_R}$

$$\therefore \frac{f_o}{k_{ro}} \Big|_{f'_{we}} = \frac{d}{df'_{we}} \left(\frac{f'_{we}}{I_R} \right)$$

that is $\frac{f_{oe}}{k_{ro}(S_{we})} = \frac{d(\frac{1}{t_0 I_R})}{d(\frac{1}{t_0})}$

Measure: Relative injectivity $I_R = \frac{\cancel{q_T M_o L}}{\cancel{\phi A L}} \frac{q_T M_o L}{\phi A L}$

Dimensionless time $t_0 = \frac{q_T t}{\phi A L}$

Fractional flow of oil in effluent $f_{oe}(t_0)$

Oil recovery rate $N_{PD} = \frac{N_p}{V_p}$

Calculate: $\bar{S}_w = S_{wi} + N_{PD}$

$$S_{we} = \bar{S}_w - t_0 f_{oe}$$

$$k_{ro}(S_{we}) = \left(\frac{d(\frac{1}{t_0 I_R})}{d(\frac{1}{t_0})} \right)^{-1} f_{oe}$$

From $k_{ro}(t_0)$, $S_{we}(t_0)$, we get $k_{ro}(S_{we})$.

$$k_{rw} = k_{ro} \frac{M_w f_{we}}{M_o f_{oe}} \quad (K_{base} = k_o @ S_w = S_{wi})$$

Range: $S_w \in [S_{wf}, 1 - S_{or}]$

Suitable conditions: 1° Capillary effects are negligible

(Can be achieved by rapid flow.)

2° Mobility ratio is unfavorable (high)