Rotating Coordinate Systems.

consider a rotating coordinate system with an angular velocity $\underline{w}$ with respect to an inertial frame.

Then, four fictitious forces have to be included when we use a rotating frame:

$$\underline{F}' = \underline{F}_{real} - \underset{\text{(Coriolis force)}}{2m\underline{w}\times\underline{v}'} - \underset{\text{(transverse force)}}{m\dot{\underline{w}}\times\underline{r}'} - \underset{\text{(centrifugal force)}}{m\underline{w}\times(\underline{w}\times\underline{r}')} - m\underline{A}_0$$

1° derivation of the formula.

$$\underline{a} = \underline{a}' + \dot{\underline{w}}\times\underline{r}' + 2\underline{w}\times\underline{v}' + \underline{w}\times(\underline{w}\times\underline{r}') + \underline{A}_0$$

where $\underline{a}$ and $\underline{a}'$ are the accelerations of a particle observed in the inertial frame and rotating frame respectively.

$\because$ $\underline{r} = \underline{r}' + \underline{R}_0$ (1)

and $\underline{r}' = x'\underline{i}' + y'\underline{j}' + z'\underline{k}'$

$$\frac{d\underline{r}'}{dt} = (\dot{x}'\underline{i}' + \dot{y}'\underline{j}' + \dot{z}'\underline{k}') + (x'\dot{\underline{i}}' + y'\dot{\underline{j}}' + z'\dot{\underline{k}}')$$

for $\left|\frac{d\underline{i}'}{dt}\right| = w \sin\langle \underline{w}, \underline{i}'\rangle$, and the direction is easy to determine.

$\therefore$ $\frac{d\underline{i}'}{dt} = \underline{w}\times\underline{i}'$

~~Similarly, we can get~~

Hence, $\frac{d\underline{r}'}{dt} = \underline{v}' + \underline{w}\times\underline{r}'$ (※)

$\therefore$ the derivative of (1) is

$$\underline{V} = \underline{V}' + \underline{w}\times\underline{r}' + \underline{V}_0 \quad (2)$$

$\therefore$ the second derivative of (1) is

$$\underline{a} = \underline{a}' + \underline{w}\times\underline{v}' + \dot{\underline{w}}\times\underline{r}' + \underline{w}\times(\underline{v}' + \underline{w}\times\underline{r}') + \underline{A}_0$$

$$= \underline{a}' + \dot{\underline{w}}\times\underline{r}' + 2\underline{w}\times\underline{v}' + \underline{w}\times(\underline{w}\times\underline{r}') + \underline{A}_0$$