Schur Product

Def : 7.5.1 Hadamard (Schur) product. $A \circ B = [a_{ij} b_{ij}]$

Prop : A, B Hermitian, then $A \circ B$ Hermitian. And

AB Hermitian $\Leftrightarrow$ A, B commute.

Lemma : 7.5.2 "rank 1 expansion"

$$A = v_1 v_1^* + v_2 v_2^* + \cdots + v_k v_k^*$$

Thm : 7.5.3 Schur product theorem

A, B positive (semi-) definite $\Rightarrow$ $A \circ B$ positive (semi-) definite

Cor : A, B Hermitian, then $\operatorname{rank}(A \circ B) \le \operatorname{rank}(A) \cdot \operatorname{rank}(B)$.

Cor : 7.5.4 Fejer's theorem

$$A \text{ positive semi-definite} \overset{A \in M_n}{\Longleftrightarrow} \sum_{i,j=1}^{n} A_{ij} B_{ij} \ge 0, \ \forall \text{ positive semi-definite } B.$$

Thm : 7.5.8 Fejer's uniqueness theorem

Cor : 7.5.9 Conditions for $[f(a_{ij})]$ to be positive semi-definite.

Thm : 7.7.9 $A^{-1} \circ B^{-1} \ge (A \circ B)^{-1}$

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