# Chapter 11, Set Operations, Image and Inverse Image.

Definition: ① Collection of Sets (Class of Sets):
a set of sets.

② Union of a Collection:
the set whose elements belong to at least one of the sets in collection $\mathcal{V}$, $\mathcal{V} = \{S_\alpha : \alpha \in A\}$.
The set is denoted by $\bigcup_{S_\alpha \in \mathcal{V}} S_\alpha$, or $\bigcup_\alpha S_\alpha$, or $\bigcup \mathcal{V}$.
In particular, if $\mathcal{V} = \{S_1, S_2, \cdots, S_n\}$, the union is denoted by $\bigcup_{i=1}^{n} S_i$

③ Intersection of a Collection:
the set whose elements belong to every set in collection $\mathcal{V}$.
The set is denoted by $\bigcap_{S_\alpha \in \mathcal{V}} S_\alpha$, or $\bigcap_\alpha S_\alpha$, or $\bigcap \mathcal{V}$.
In particular, if $\mathcal{V} = \{S_1, S_2, \cdots, S_n\}$, the intersection is denoted by $\bigcap_{i=1}^{n} S_i$

④ Complement of B in A:
$A \setminus B \equiv \{x : x \in A, x \notin B\}$

⑤ Image of S under f:
$f(S) \equiv \{f(x) : x \in S\}$, where $f: A \to B$, $S \subseteq A$.

⑥ Inverse Image of C under f:
$f^{-1}(C) \equiv \{x : f(x) \in C\}$, where $f: A \to B$, $C \subseteq B$.

Properties: ① If $S_\alpha \subseteq T_\alpha$ for all $\alpha$, then $\bigcup_\alpha S_\alpha \subseteq \bigcup_\alpha T_\alpha$, $\bigcap_\alpha S_\alpha \subseteq \bigcap_\alpha T_\alpha$;

② $R \setminus (R \setminus S) = S$;
if $A \subseteq B$, then $C \setminus B \subseteq C \setminus A$, for every $C$;

③ Distributive Law:
$A \cap (\bigcup_\alpha S_\alpha) = \bigcup_\alpha (A \cap S_\alpha)$, $A \cup (\bigcap_\alpha S_\alpha) = \bigcap_\alpha (A \cup S_\alpha)$;

④ de Morgan's Law:
$A \setminus (\bigcup_\alpha S_\alpha) = \bigcap_\alpha (A \setminus S_\alpha)$, $A \setminus (\bigcap_\alpha S_\alpha) = \bigcup_\alpha (A \setminus S_\alpha)$;

⑤ $f: A \to B$, if $U \subseteq V \subseteq A$, then $f(U) \subseteq f(V)$;
if $X \subseteq Y \subseteq B$, then $f^{-1}(X) \subseteq f^{-1}(Y)$;

⑥ $f: A \to B$, $U \subseteq f^{-1}(f(U))$, and $f(f^{-1}(X)) \subseteq X$;

⑦ $f(\bigcup_\alpha S_\alpha) = \bigcup_\alpha f(S_\alpha)$, and $f^{-1}(\bigcup_\alpha S_\alpha) = \bigcup_\alpha f^{-1}(S_\alpha)$;

⑧ $f(\bigcap_\alpha S_\alpha) \subseteq \bigcap_\alpha f(S_\alpha)$, and $f^{-1}(\bigcap_\alpha S_\alpha) = \bigcap_\alpha f^{-1}(S_\alpha)$;

⑨ If $X \subseteq A$, then $f(A \setminus X) \supseteq f(A) \setminus f(X)$,
If $Y \subseteq B$, then $f^{-1}(B \setminus Y) = f^{-1}(B) \setminus f^{-1}(Y)$.