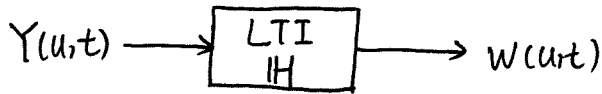
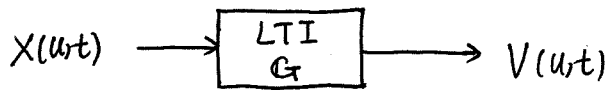


W.S.S. - LTI Relationship (Chap. 15.2)



Suppose $X(u,t)$ and $Y(u,t)$ are jointly w.s.s., defined on the same \mathcal{U}, \mathcal{T} .

G, H are stable LTI operators.

Then $V(u,t)$ and $W(u,t)$ exist in the m.s.s. :

$$V(u,t) = (g * X)(u,t), \quad W(u,t) = (h * Y)(u,t)$$

Since $V(u,t), W(u,t)$ are respectively w.s.s., and their cross-correlation R_{VW^*} is a fn.

$$\begin{aligned} R_{VW^*}(t_1, t_2) &= E[(g * X)(u, t_1) \cdot (h * Y)^*(u, t_2)] \\ &= (g * R_{XY^*} * h^*)(t_1, t_2) \\ &= (g * R_{XY^*} * h^*)(\tau) \end{aligned}$$

$\therefore V(u,t), W(u,t)$ are jointly w.s.s.

Using cross-power spectral density, we have the following w.s.s.-LTI relationship

$$\begin{cases} S_V(f) = |G(f)|^2 S_X(f), & m_V = G(0) m_X \\ S_{VX^*}(f) = G(f) S_X(f) \\ S_{VW^*}(f) = G(f) S_{XY^*}(f) H^*(f) \end{cases}$$

Note: 1° ~~power~~ ("convolution thm")

$$\mathcal{F}\{g * R_{XY^*} * h^*\} = \mathcal{F}\{g\} \cdot \mathcal{F}\{R_{XY^*}\} \cdot \mathcal{F}\{h^*\}$$